Industrial Policy with Development Characteristics: Fertilizer Subsidies in Times of Crisis*

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Abstract

We study time-consistent optimal policy when undiversified owner-operators face financial frictions and a planner with limited instruments. We apply it to fertilizer subsidies, one of the largest sector-specific policy instruments in the developing world and whose price is becoming increasingly volatile. We collect household-level data from 444 rural Rwandan villages from 2020 - 2024 and exploit the doubling of fertilizer prices after Russia's invasion of Ukraine. Relative to less fertilizer-intensive villages, more fertilizer-intensive villages experience 30 percent lower fertilizer spending, 21 percent lower harvests, and 11 percent higher output prices. These patterns discipline key model elasticities. Quantitatively, the optimal subsidy remains roughly constant at 10 percent before and after the shock because productive reallocation toward less fertilizer-dependent villages lowers the optimal subsidy, while poverty alleviation in the harder-hit fertilizer-intensive villages raises it by a similar amount.

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1 Introduction

In many low-income countries, governments pursue economic development with only a small set of sector-specific tools. The ubiquity of small-scale, un-diversified household-firms and difficulty tracking employment statuses often requires production-side policies to serve a dual role as social protection. This paper asks how a policymaker should balance these competing objectives.

We study this question in the context of fertilizer subsidies. Fertilizer subsidies are among the most common industrial policies in developing countries, reflecting agriculture's centrality in development (Gollin, 2023) and fertilizer's role in its labor productivity (Restuccia et al., 2008; Boppart et al., 2023).[1] At the same time, near-complete reliance on imported fertilizer ties these economies to global energy markets and exposes them to increasingly frequent price shocks in the current geopolitical environment. Russia's invasion of Ukraine, for example, tripled global fertilizer prices and brought fertilizer policy back to the forefront of policy debates in developing countries.[2]

We build a dynamic general equilibrium model of household-farmers subject to limited borrowing and financial frictions, similar to canonical models reviewed in Buera et al. (2015a) and Kaboski (2023), but extended in several dimensions. Households live in villages that produce differentiated agricultural products, which are then aggregated across villages. Each village is defined by two additional dimensions of heterogeneity: its technology, which governs fertilizer's importance in production, and its financial friction, which determines households' ability to borrow to fund agricultural production. The model therefore allows two empirically relevant reasons for low fertilizer use, each with distinct policy implications.[3] This technological heterogeneity, and its interaction with policy, is central to our paper.

We study the decision of a Ramsey planner who cannot commit to future policy and therefore focus on time consistent optimal policy. She has two instruments: *ex post* village-specific lump sum transfers and a fertilizer subsidy. Guided by observed policy in developing countries, the subsidy must be uniform across villages. These instruments help separate production and consumption motives in optimal policy. In the quantitative analysis, we

[1] These subsidies make up 25 percent of agricultural budgets in many Sub-Saharan African countries (Jayne et al., 2018; Holden, 2019) and are valuable enough to be granted special exception for developing countries in global trade treaties by the World Trade Organization (Section 6, Agreement on Agriculture, World Trade Organization (2025)).

[2] Russia's invasion prompted over 700 policy responses worldwide (Amaglobeli et al., 2023), with no shortage of views on how fertilizer policy should respond. Former World Bank president David Malpass called for a renewed focus on fertilizer industrial policy in Sub-Saharan Africa (Malpass, 2022). Former head of the U.N. World Food Programme David Beasley warned of "hell on earth" if fertilizer prices were not dealt with properly (Associated Press, 2022). The African Development Bank called for "urgent countercyclical policy response such as subsidies to mitigate the impact of higher food and energy costs (African Development Bank Group, 2022)." Individual countries adjusted policy as well and we will discuss the Rwandan context in detail in this paper.

[3] Marenya and Barrett (2009) and Theriault et al. (2018) show that soil characteristics affect the yield response to fertilizer even within small geographic regions of Kenya and Burkina Faso. Beaman et al. (2023) highlights large returns to agricultural credit in Mali. We provide additional evidence from Rwanda in Section 2.

study the case in which the planner has only the subsidy available and must balance both objectives. Our main exercise asks how the optimal policy should change after a fertilizer price shock like the one observed in 2022.

We first characterize the planner's decision in a static version of the model that isolates the central trade-offs. When the planner has access to both *ex post* transfers and a subsidy, the model admits a simple qualitative policy rule: if the fertilizer price rises, the optimal subsidy falls if villages produce substitutes, and rises if they produce complements. This holds for any distribution of financial frictions and technological heterogeneity, as long as the marginal technology distribution is not degenerate.

The intuition rests on how equilibrium prices and production shares adjust after the shock. When villages produce complements, equilibrium prices adjust to compensate the weak link – here, fertilizer-intensive villages hit hardest by the shock – echoing the logic from the literature on sectoral linkages (e.g. Jones, 2011). When villages produce substitutes – as we will find – the opposite occurs. Prices shift production away from the most affected villages. In either case, financial frictions impede efficient reallocation, leaving the planner to accomplish it. Technological heterogeneity makes this possible. Because fertilizer cost shares differ across villages, a uniform subsidy changes relative productive incentives. Therefore, shifting production toward fertilizer-intensive villages requires raising the subsidy, away requires lowering it. Optimal policy operates through reallocation and its direction depends on how villages aggregate.

This simpler model offers two additional insights for quantification. First, technological heterogeneity is central. Abstracting from it implies that the planner does not adjust optimal policy in response to a fertilizer price shock, regardless of the distribution of financial frictions. This introduces a normative concern to a classic misallocation issue. We observe dispersion in fertilizer use, but attributing all of it to financial frictions would shut down the key policy channel in our model. This would be the implication, for example, of assuming identical production technologies across villages. Recent work highlights the importance, and difficulty, of distinguishing these channels in agriculture (Gollin and Udry, 2021).

Second, the sharpness of this result depends crucially on the planner's ability to decouple production from consumption through *ex post* transfers. These transfers diversify otherwise undiversified farm owner-operators, in the same way assuming a representative household would. In reality, weak institutions often make this difficult. In this case, the planner must balance productive reallocation against the consumption losses it creates.

Some version of this argument is at the heart of most policy recommendations after the 2022 fertilizer price shock: marginal utility is high, and therefore subsidizing production helps cushion consumption. But again, the central role played by reallocation here highlights why this partial equilibrium thinking is incomplete. Raising the optimal subsidy is not about poverty levels, but how poverty co-moves with the general equilibrium reallocation of production after the shock. The single subsidy balances both and determines who bears the consumption loss, and requires understanding not only the level of marginal utility, but how changes in marginal utility co-vary with technology across space after the shock.

Both issues necessitate careful measurement when we take the model to the data. For this, we turn back to our full dynamic model. To capture relevant smoothing channels, we allow households to respond to fertilizer prices along several dimensions. Households can switch technologies, moving between a fertilizer-intensive technology and a labor-only alternative. This assumption is motivated by a large micro-development literature documenting high extensive margin fertilizer adjustment (Dercon and Christiaensen, 2011; Suri, 2011), which we also confirm in our data. Even within the fertilizer technology, households can substitute fertilizer for labor within a CES production structure. They can also leave the farm to work in the non-agricultural sector.

Our second contribution in this paper is to offer a serious quantification of this model, measuring the smoothing channels discussed above along with the relative importance of heterogeneity in technology and financial frictions. Doing so requires detailed micro data and variation that identifies key elasticities. We bring the model to the data in Rwanda, where we collect a household-level panel dataset from 2020 - 2024 for 15,000 households across 444 villages. Russia's 2022 invasion of Ukraine caused real fertilizer prices in Rwanda to double. The government responded by raising the subsidy from 30 to 45 percent, causing the program to grow from 4 to 19 percent of the Agriculture Ministry's budget between two seasons.[4] Taking the global shock and government response in tandem, Rwandan farmers paid 70 percent more per kilogram of fertilizer in 2022 than they did one season before. Professional price forecasts confirm this change was unexpected. This gives variation useful for identification along with the disaggregated data needed to study it.

In shift-share regressions, we find large and statistically significant changes in many dimensions. Farmers in villages with higher baseline fertilizer decrease both fertilizer use and harvests relative to less exposed villages. These villages experience 30 percent lower fertilizer spending and 21 percent lower harvests. At the same time, these villages become more labor intensive, with earnings outside the village falling by nearly 40 percent, shifting income toward agricultural income (both farm profit and wages). Output prices rise by 11 percent and agricultural wage by 24 percent, consistent with labor being pulled back into

[4] These same patterns hold in several other countries. Burundi raised their subsidy from 40 to 60 percent in 2022. Ghana moved to eliminate their cocoa fertilizer subsidy in 2019, only to resume it in 2023. Kenya re-introduced a general fertilizer subsidy previously scrapped in 2020. Tanzania implemented an explicitly temporary fertilizer subsidy in 2022. Zambia held farmer prices fixed in 2022, implicitly raising the subsidy level.

agriculture after the shock.

We then make use of the richness of our data to calibrate the model. Several key model parameters can be recovered from the cross-equation restrictions made up of these same shift-share coefficients. Casting the model in time differences offers a substantial simplification of the relationship between model and data, borrowing intuition from exact hat methods in trade (see Caliendo and Parro, 2022, for a recent review). This lets us quantify the forces emphasized above, including changes in model-consistent distortions in the spirit of Restuccia and Rogerson (2008) or Hsieh and Klenow (2009), while also exploiting the general equilibrium price variation induced by the shock.[5]

With the calibrated model in hand, our first quantitative exercise asks how optimal policy evolves when fertilizer price spikes, as it did in 2022. We find that the optimal subsidy remains essentially unchanged at 10 percent before and after the shock. The rationale is two-fold. First, villages produce substitutable products, which incentivizes reallocating production away from fertilizer-intensive villages, thus pushing the optimal subsidy down. Second, villages are sufficiently poor that the planner cushions their consumption less, which pushes the subsidy up. The net change in the subsidy is small not because households are well-insured against the shock, but the opposite.

We conduct a series of counterfactuals to isolate model features that lead to this result. In the first, we remove the ability of households to shift into the labor-only technology. Without this outside option, lowering the subsidy has larger consumption consequences. The optimal subsidy becomes higher and more responsive to the shock, rising from 37 to 45 percent. This follows from the fact that we match the large extensive margin fertilizer adjustment across villages observed in the data, along with the productivity consequences of not using fertilizer. Matching these patterns also implies that we estimate a nearly Cobb-Douglas fertilizer-intensive technology. Together, they generate an aggregate own-price elasticity of fertilizer that is much larger than one would infer from extrapolting household-level intensive-margin responses. The result is consistent with cross-country evidence in Boppart et al. (2023), and we offer a micro-founded and policy-relevant interpretation of their importance.

Our second counterfactual is motivated by the gap between the model-implied optimal subsidy and observed policy, where the government increased the subsidy from 30 to 45 percent. We ask whether observed policy can be rationalized as optimal in an economy that looks more like Rwanda in the early 2000s: poorer and with a lower return to savings. In this case, the optimal subsidy rises from 31 to 46 percent, matching the data. This model delivers

[5] In contrast, recent work in the same methodological macro-development tradition tends to use "small scale" RCTs to isolate parameters without the complication of general equilibrium effects. See for example Buera et al. (2021), Fujimoto et al. (2023), VanVuren (2025), and Falcao Bergquist et al. (2025), among others reviewed in Buera et al. (2023). Measuring these equilibrium forces is central to our strategy.

an evolution of distortions becomes too similar across villages. This result is consistent with recent policy deliberations in Rwanda. Interestingly, this result is consistent with recent policy deliberations in Rwanda, acknowledging the over-subsidization but also the difficult political economy questions that arise when attempting to lower it (World Bank, 2025).

Motivated by this result, our last exercise measures the optimal transition path from the observed baseline of 30 percent to the eventual optimal level of 10 percent. The model predicts a sharp, almost immediate decline to 10 percent. The welfare gains vary across villages, but are large enough that each village would prefer the Russia-induced price shock if it also came with the subsidy decline, compared to having no shock but a subsidy held at 30 percent. Overall, including the transition, optimal policy generates 0.9 percent consumption-equivalent welfare gains compared to the observed subsidy evolution.

1.1 Related Literature

In addition to literature discussed above, this paper joins a growing literature focused on the role of intermediate inputs in the agricultural sector and related policy (Diop, 2023; Garg and Saxena, 2023; Ghose et al., 2024). Most closely related in this dimension is interesting recent work by Mazur and Tetenyi (2024) and Chakraborty et al. (2025), both of whom study the impact of fertilizer subsidy programs (along with other policies) in general equilibrium models. Our focus on understanding how a subsidy should evolve with its corresponding input price is new to this literature and shows that financial frictions alone are not generally the key driver of the optimal subsidy price schedule.

As a modeling device, our model resembles those used to study the real implications of credit crunches with financial frictions (Buera et al., 2015b; Buera and Moll, 2015), though we tailor it in various ways to the specifics of our more developing-country implementation. Our normative perspective on optimal policy away from steady state (as in Itskhoki and Moll, 2019) highlights important caveats and provide guidance on both the sign and magnitude of policy design in response to global input price shocks. Centrally, we highlight the importance of technological heterogeneity on qualitative and quantitative properties of optimal policy. These insights apply more broadly to these models as well.

Finally, central to our results are how general equilibrium effects induce distributional consequences that are central to policy. For positive perspectives with a focus on developing countries, see also Rotemberg (2019) and Falcao Bergquist et al. (2025), who build models that isolate the importance of productive reallocation or consumption redistribution exclusively, and Manysheva (2022) on the interaction of financial frictions with incomplete land markets. A key point here is that these gains may not be enough to overcome the cost of financing the program, as highlighted in the education market by Fujimoto et al. (2023).

2 Setting, Data, and Fertilizer Price Shock

Global inorganic fertilizer production is concentrated in a small number of countries because of comparative advantage in key inputs like natural gas. Russia is the largest exporter globally, implying that shocks to Russia's ability to export play an important role in global price fluctuations. In this section, we detail how global fertilizer prices translated into changes in Rwandan policy. Rwanda offers a useful setting for several reasons related to the generalizability to other contexts. First, agricultural value added per worker is only half of GDP per capita, a labor productivity gap found in many other developing countries (Kruse et al., 2023; Herrendorf et al., 2022). Second, Rwanda imports the entirety of its inorganic fertilizer and the government is heavily involved in setting the subsidy level. Third, the policy responds directly to the shock described below.

2.1 Data

Our main data source for the empirical results is primary data. We collect household data from 444 villages across most of Rwanda. This dataset was originally designed to measure the impact of an RCT on last-mile infrastructure and as such covers all of Rwanda except the relatively flat east part of the country.[6] The dataset began in 2020 with a baseline and has been collected annually since, implying 5 visits per household (2020 – 2024, inclusive). We exclude all treated villages from the RCT and focus exclusively on control villages. Despite our description as a household-level panel, these data are an individual-level panel. We collect household rosters and ask individual-level questions on demographics, earnings, and health and education outcomes, along with household-level questions about farming practices, savings, and other related decisions.

In the Appendix, we compare our data to nationally representative data in Rwanda. Our households are similar in demographics and land-holdings, with an average farm size of 0.33 hectares in both. Our sample is slightly more likely to use fertilizer, though the slope with land size is equal across the two datasets.

2.2 Fertilizer Market Structure in Rwanda

The main fertilizers used in Rwanda all include nitrogen. These include DAP (a combination of nitrogen and phosphorus), a balanced nitrogen-phosphorus-potassium (NPK) 17-17-17,

[6]See Macharia et al. (2022) for the pre-analysis plan and other details about the RCT.

and urea (primarily nitrogen). These three fertilizers make up 90 percent of the value of all fertilizers used in Rwanda in 2020 (National Institute of Statistics Rwanda, 2021). Nitrogen-based fertilizers use ammonia to deliver the nitrogen to plants, which is produced primarily with natural gas.[7] The centrality of natural gas in the production process implies two relevant features. First, shocks to natural gas prices will affect fertilizer prices. Second, countries with natural gas have a comparative advantage in production. Because of this, production is concentrated in a handful of countries and over 90 percent of demand in Sub-Saharan Africa – and 100 percent in Rwanda – is met via imports.[8]

Market Structure Fertilizer is not freely traded in Rwanda, instead managed by the Ministry of Agriculture and Animal Resources (MINAGRI). The government offers licenses for organizations to import fertilizer from abroad (usually 4 – 10 private firms and large NGOs), who then sell fertilizer via a set of licensed retailers scattered throughout the country. The government sets two prices: the market price and the subsidy level. The market price is the price received by the importers from the national government, and roughly tracks international prices. Technically, importers sell to retail vendors at a subsidized rate and are reimbursed by the government. Farmers then pay the market price net of the subsidy.[9,10] The normal price-setting involves the government announcing prices on July 1 preceding fertilizer purchases for the main growing season, then holding it fixed for the crop year, resetting in the next July. We say "normal" here because we will shortly show how price announcements adjust after the 2022 fertilizer price shock.

Farmer Access to Subsidies By 2020, the fertilizer subsidy was universally available. Ndushabandi et al. (2018) and Spielman et al. (2023) offer detailed histories of the program, but we discuss some key points that motivate modeling decisions here.

The fertilizer subsidy program grew out of the Rwandan Crop Intensification Program started in 2007. At the start, the program was targeted explicitly. A farmer was required to grow at least one hectare of 6 priority crops and, in return, was given a coupon for up to

[7]Ammonia (NH_3) is easier for plants to break down than the nitrogen in the air (N_2) due to the triple-covalent bond between the two nitrogen atoms. The Haber-Bosch process subjects natural gas to pressure, causing its hydrogen atoms to bind with nitrogen atoms in the air. The induced reaction, $N_2 + 3H_2 \rightleftharpoons 2\ NH_3$, creates the ammonia used in fertilizer. Synthesizing ammonia accounts for 3-5 percent of global natural gas consumption (Song et al., 2018).

[8]The only country that produces fertilizer domestically is Nigeria, which produces extremely small urea quantities. Interestingly, nearly all of this is exported off the continent.

[9]This process is managed by the Rwandan government's Smart Nkunganire System (link here). There are no subsidized products sold outside this system, which helps facilitate repayment.

[10]One natural question when dealing with non-market prices is the equilibrium possibility of quantity restrictions. In practice this seems to be of limited concern, as farmers can almost always access the fertilizer they demand. By law, any fertilizer importer is required to service all agro-dealers in the country. Thus, the fertilizer market is national instead of a collection of closed local fertilizer markets, limiting the possibility of local shocks driving a local mismatch of supply and demand. Second, even when there is the possibility of national under-supply, the government induces imports to correct the projected imbalance. Both our farmer survey evidence and discussions with importers and agro-dealers support the view that this margin is relatively unimportant, though could play an indirect role through the government's budget.

two 50-kg fertilizer bags. By 2013, there was growing recognition that targeting was limiting take-up. Three large changes were put in place. First, additional crops were added so that every district in Rwanda had at least one crop covered. The subsidy now covers fertilizer for maize, beans, wheat, soybeans, rice, potatoes, cassava, bananas, vegetables and fruit trees, covering essentially all production in our dataset except coffee. Second, the mono-cropping requirement was eliminated in response to farmer concerns about diversification. Third, the coupon system was replaced with a flat subsidy.

Together, these changes implied near universal eligibility by 2015. Remaining ineligible farmers are mostly due to self-selection into not registering for the system. Anticipating the model, we will assume universal coverage of the subsidy for these reasons.

2.3 Shock to Global Fertilizer Prices and Response in Rwanda

The reliance on imports exposes countries like Rwanda to global shocks, especially those that affect countries like Russia. In 2020, Russia produced 32 percent of the world's NPK and 10 percent of urea, contributing 13 percent of all nitrogen exports in the world (FAOSTAT, 2024).

In 2022, the Russia-Ukraine war created two main issues that caused fertilizer prices to spike. First, fertilizer and ammonia exports declined, the latter of which made it difficult to process fertilizer in other, non-natural gas producing countries like Morocco. Recent estimates suggest Russian ammonia exports fell 63 percent between 2021 and 2022, and urea fell 23 percent (Glauber and Laborde, 2022). While Rwanda primarily imports fertilizer from Morocco and Saudi Arabia, the decline in global supply impacted global prices.[11] The second implication of the war was its impact on the natural gas market. Russian natural gas and oil exports fell in 2022, with Europe attempting to substitute Russian oil and gas with U.S. natural gas after exiting a cold 2021 winter with limited gas reserves (EIA, 2023; Gil Terte, 2023). This redirected American gas exports toward Europe for mostly non-fertilizer uses. Thus, not only was fertilizer in relatively short supply globally, its most important input was increasing in price for non-agricultural reasons. In Appendix A we show that monthly global fertilizer prices closely track natural gas prices.

The relative (but not total) importance of these shocks remains subject to debate, but clearly none are related to Rwandan policy decisions. Figure 1 shows they were also unanticipated. It plots the global prices of DAP and urea from 2015 to 2023 using the World Bank Commodity Price Outlook (World Bank, 2024). The nominal price shows the sharp rise from

[11] Morocco is the world's largest phosphate producer, thus an important country for DAP exports, which delivers nitrogen and phosphate. In 2020, 58 percent of Rwanda DAP imports came from Morocco, 28 percent from Saudi Arabia and 14 percent from Russia. By 2022, Russian DAP fell to zero and was partially made up for by an increase in Saudi Arabian fertilizer. The total quantity of Rwandan DAP imports fell from 24 million tons in 2020 to 15.6 million tons in 2022.

a period of relative stability, with DAP and urea rising to 2.5 and 3 times their 2020 level by 2022.[12] The dashed lines in Figure 1 plot the forecasted prices from each October release from 2018 to 2022. The 2018 - 2020 forecasts of continued stable prices were not realized.

Figure 1: Global Fertilizer Prices and Projected Prices (Current USD/metric ton)

(a) DAP (b) Urea

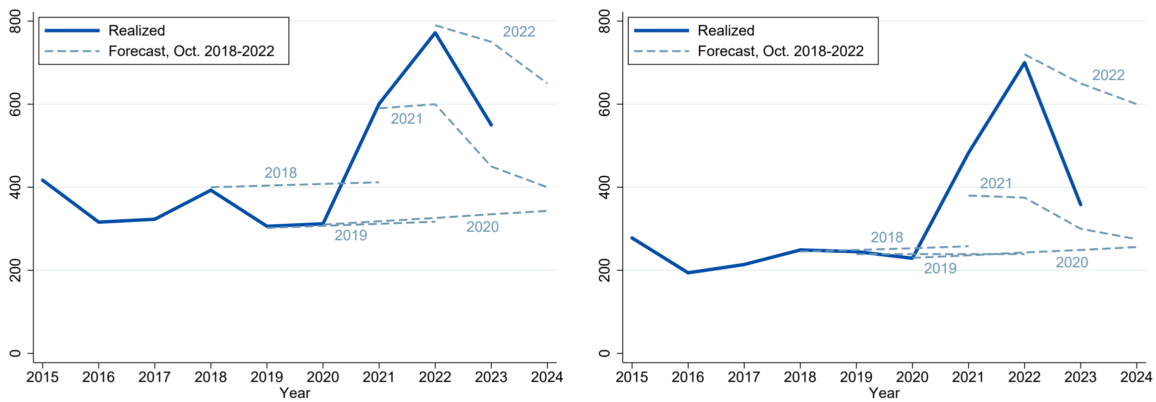

Figure Notes: The solid line plots the realized price for DAP and urea from 2015 - 2024. The dashed lines plot the projected prices each year. Both realized and projected prices are derived from the World Bank Commodity Price Outlook (World Bank, 2024).

2.3.1 Response in Rwanda

The Rwandan government historically updated fertilizer prices once per year in July, which was before the start of one of the two seasons in Rwanda ("Season A").[13] This policy-induced price stickiness was irrelevant when global fertilizer prices were stable during most of the 2000s. But higher global prices forced an unprecedented off-schedule price change in January 2022 before the start of Season B. The market price for DAP was adjusted upward by 40 percent. For some sense of scale, the average annual price change between 2018 and 2021 was 4 percent. Similar adjustments were made for urea and NPK.

In Figure 2, we use government documents to plot the government-defined market price for 3 main fertilizers along with the subsidy rate. Farmers pay (1-subsidy) × the market price. The figure also includes markers at any point when the government releases an updated price (even if that price is identical to the previous one). The discussion above is reflected

[12] The fertilizer price actually starts to rise in mid-2021. This was due to an issue of low U.S. coal inventories putting upward pressure on natural gas prices in 2021. That pressure was released at the end of 2021 on the backs of higher gas production and a mild winter (EIA, 2022). Thus, the proper non-war counterfactual would have been a temporary and relatively minor increase in natural gas prices (and thus fertilizer prices) in 2021 only. This is consistent with the 2021 forecast in Figure 1.

[13] The agricultural calendar in Rwanda is made up of two main cropping seasons. Season A begins with planting in September and ends with harvesting in January and February, while Season B runs from March to June. These seasons are generally referenced by the year of Season A's harvest, so that the 2022 season consists of Season A from September 2021 – February 2022 and Season B from March 2022 – June 2022.

in these figures. First, prices and subsidy rates remain stable until 2022. Second, through 2021, all price adjustments occur in Season A and are held fixed in B. However, in 2022-B, both the market price and subsidy rates are updated due to the stark rise in international prices.

Figure 2: Rwandan Fertilizer Prices

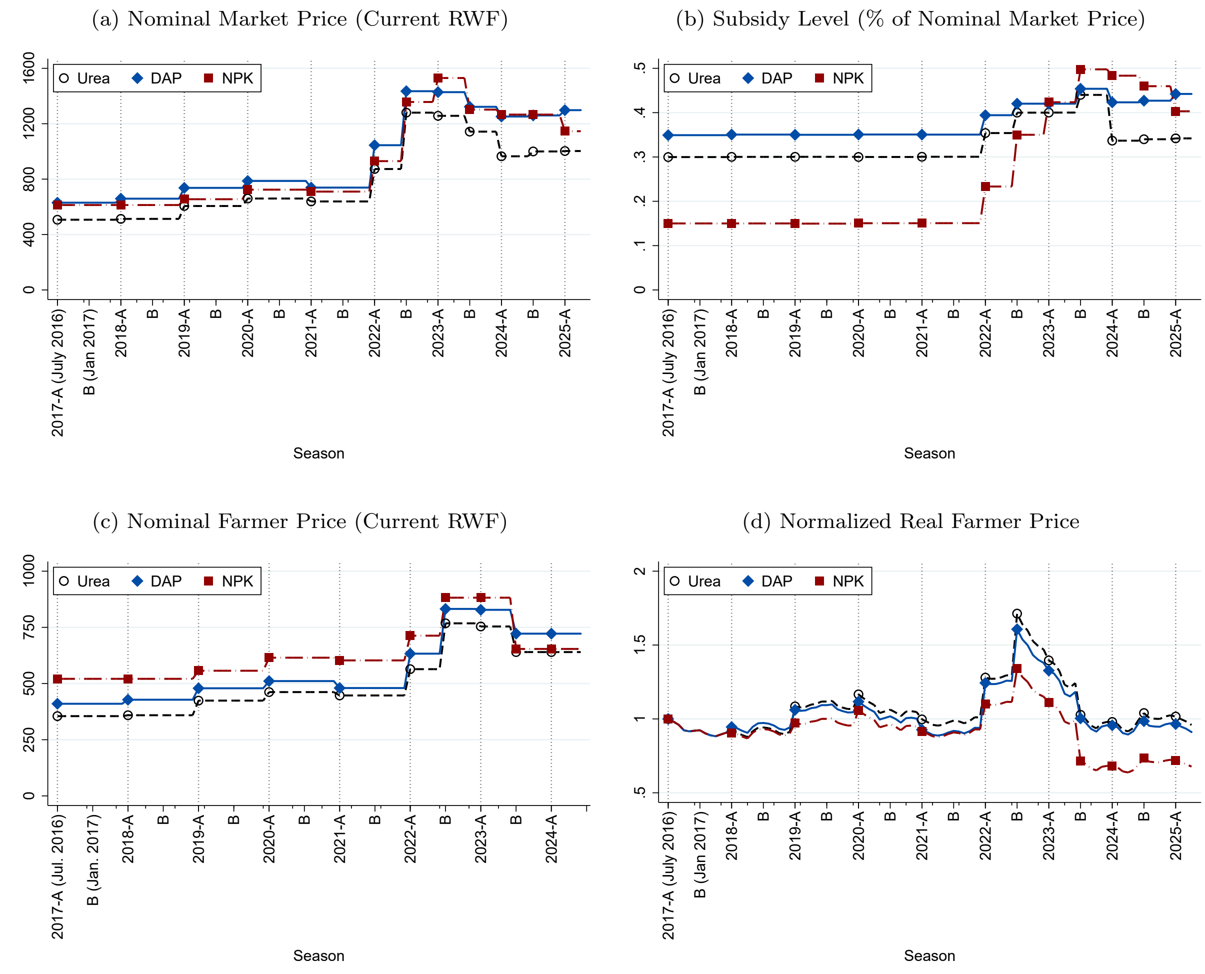

Figure Notes: Panel (a) plots the nominal market price defined by the Rwandan government in current Rwandan francs. Panel (b) plots the subsidy level. Panel (c) is the nominal farmer price, equal to (1-Subsidy) × Nominal Market Price. Panel (d) plots the nominal farmer fertilizer price deflated by the rural Rwandan CPI and normalizes by the 2017-A prices. In all figures, a marker is included when new prices are released even if the price is not updated. Notice that before 2022, all price updates were in Season A.

The government made two adjustments in 2022. The first was an increase in the market price (Figure 2a). Second, they limited pass-through to farmers by increasing the subsidy rate (Figure 2b). As such, program jumped from 4 to 19 percent of MINAGRI's total budget; from 5 billion francs to 13 billion. Figure 2c then plots the implied nominal prices for farmers. To summarize, Figure 2d plots the relevant shock: the farmer fertilizer price deflated by the rural Rwandan CPI, normalized at 2017-A.

In Appendix A, we use data from traders to show that our data show the same patterns (as should be expected, given that we survey the same retailers). Furthermore, we offer evidence that prices of other intermediate inputs, like fungicide and insecticide, do not show the same price increase.

3 Empirical Results

To estimate the impact of this shock across villages, we take advantage of the fact that villages differ substantially in their fertilizer intensity, measured here as total village fertilizer expenditure per acre of land in agricultural production in our 2020 baseline. We focus on village-level intensity instead of farmer-level intensity due to general equilibrium effects that will become clear below. The fertilizer intensity of village v at survey wave t is

$$\widetilde{f}_{vt} = \log\left(\frac{\sum_{h \in v} f_{hvt}}{\sum_{h \in v} \ell_{hvt}}\right).$$

where f_{hvt} is the fertilizer expenditures of household h in village v at year t. ℓ_{hvt} is land used in production. There is a substantial amount of variation in this moment at baseline, with a difference between fifth and ninety-fifth percentile village of 3.47 log points (from -1.75 to 1.72). We then consider the following regression

$$y_{hvmt} = \alpha + \beta Post_t + \eta\left(Post_t \times \tilde{f}_{v0}\right) + \delta_m + \theta_v + \zeta_t + \varepsilon_{hvmt}, \tag{3.1}$$

Here, y is the outcome for household h in village v in month m in year t. $Post_t$ equals one after January 2022, and we include village, month, and survey wave fixed effects.[14] Our interest here is the parameter η, which measures how outcomes vary with baseline fertilizer intensity $\widetilde{f}_{v0}$ after January 2022.

In all cases, we trim continuous outcomes at 1 percent and run the regressions as Poisson regressions to interpret as percentages with zeros. Outcomes in Rwandan francs are deflated by the monthly Rwandan CPI for rural consumers.

3.1 Income Sources

Table 1 covers the main income sources for households in the village. Panel A begins with farming outcomes.

Column (1) shows that fertilizer use declines more in places with higher baseline usage,

[14] Because of the number of villages covered by our study, survey waves are conducted over multiple months.

Table 1: Income Generating Activities

	Fertilizer		Harvest Value		
Panel A: Agriculture	Expenditure (1)	Use Any (2)	Total (3)	Cash Crops (4)	Staple Crops (5)
Post × Village Fertilizer Intensity	-0.089***	-0.059***	-0.060***	-0.050**	-0.065***
	(0.020)	(0.013)	(0.015)	(0.021)	(0.016)
Observations	40,070	40,070	40,456	40,456	40,456
Panel B: Wage Earning	Total (6)	Outside Village (7)	Inside Village (8)		
Post × Village Fertilizer Intensity	-0.049***	-0.108***	0.014		
	(0.017)	(0.028)	(0.026)		
Observations	39,997	39,978	39,999		

Table notes: Standard errors clustered by village are in parentheses. The regression is given in (3.1) and is run as a Poisson regression. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and ***.

showing that these villages are indeed more exposed to the shock (along with the mechanical aspect of the result that it bounded by zero below). Making use of the 3.47 log point 95/5 difference to get a sense of magnitude, the decline in fertilizer expenditures is about $-0.089 \times 3.47 = 31$ percentage points larger between the top and bottom of the distribution. Column (2) shows that much of this comes on the extensive margin, with the likelihood of using any fertilizer declining by 20 percentage points more at the top of the village fertilizer distribution. This decline in fertilizer impacts harvest. Column (3) shows that a one log point increase in baseline fertilizer intensity causes a 6 percent decline in harvests. Both cash and staple crops exhibit similar variation across villages.

Panel B covers total household labor market earnings, a second critical income source. Again, fertilizer intensive villages exhibit a much larger decline in wage income, entirely accounted for by a decline in wage earnings outside the village. Thus, the shock draws workers back into the farming sector.

3.2 Price Changes

The results above suggest a retrenchment of labor-intensive farm activity after the shock. We show that this is consistent with general equilibrium forces. We start with crop prices using a crop-household level regression

$$p_{chvmt} = \beta Post_t + \eta \left(Post_t \times \tilde{f}_{v0} \right) + \delta_{cm} + \theta_v + \zeta_t + \varepsilon_{chvmt},$$

where c is the crop and the remaining subscripts are as before, except we replace month fixed effects with crop-month fixed effects to take into account crop-specific seasonal price patterns. Table 2, Column (1) shows that the price of agricultural output rises after the shock.

Table 2: Prices

VARIABLES	Crop Prices (1)	Daily Wage (2)
Post × Village Fertilizer Intensity	0.032***	–0.028**
	(0.006)	(0.011)
Post × Inside		–0.082***
		(0.017)
Post × Village Fertilizer Intensity × Inside		0.068***
		(0.0015)
Obs. Level	Crop-HH	Indiv
Observations	59,145	24,854

Table notes: Standard errors clustered by village are in parentheses. All outcome variables are in logs and regressions are run as OLS. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and ***.

Our second set of prices are wages. Here, defining $\mathbb{1}^{\text{inside}}_{ivmt} = 1$ if individual i's job is inside her village v in month m at wave t, we run the individual-level regression

$$\begin{aligned} w_{ivmt} &= \alpha Post_t + \beta \left[Post_t \times \mathbb{1}^{\text{inside}}_{ivmt} \right] + \eta \left(Post_t \times \tilde{f}_{v0} \right) + \gamma \left(Post_t \times \tilde{f}_{v0} \times \mathbb{1}^{\text{inside}}_{ivmt} \right) \\ &\quad + X_{ivmt} + \delta_m + \theta_v + \zeta_t + \varepsilon_{ivmt} \end{aligned}$$

where w is the daily wage and controls include age, age squared, and indicators for education level (primary, secondary, vocational, tertiary). Thus, $\hat{\eta}$ measures the average effect and $\hat{\gamma}$ measures any differential effect for agricultural jobs in the village. These results are in Column (2) of Table 2. High fertilizer intensive villages see rising village wages relative to outside wages.[15]

To recap, villages using more baseline fertilizer (1) decrease fertilizer use and harvests, (2) see higher sales prices for crops, (3) see within-village agricultural wages rise, and (4) income shares shift toward the agricultural sector.

3.3 Additional Results in the Appendix

In Appendix A, we provide several additional results. First, we show that other intermediates like fungicide and insecticide do not change prices in the same way that fertilizer does. This acts as a placebo test because these intermediates are more readily available and produced in more countries and thus should not be subject to the same shock. Second, we show that the price effects observed here are driven by villages that are "large" in the markets in which they trade. Third, we use import and export data to show that Rwanda is importing primarily

[15] While we do not ask households for a detailed price listing of non-agricultural purchases (only aggregated expenditures), we do collect those data from market traders. The two main non-agricultural products sold in local markets are cooking oil and salt. There is no change in the price of either. Thus, the price changes here should be interpreted as changes in the relative price.

sunflower oil from Ukraine but little actual food, implying that a direct food supply shock is unlikely to drive these results. In total, Ukraine accounts for 0.6 percent of agricultural imports into Rwanda, almost all of which is sunflower oil. As a complement to this result, we use our own data to show that prices rise substantially on crops that are entirely produced and consumed locally in Rwanda, implying an important role for domestic productivity.

3.4 "Exposure" to Shock and Underlying Mechanisms

The remaining issue here is that exposure – fertilizer expenditures per acre – is endogenous. It is driven by different underlying fundamentals that respond to policy. On one hand, if villages have heterogeneous technologies, high baseline fertilizer use is indicative of greater exposure to the fertilizer price shock. On the other hand, if villages instead differ in frictions like credit constraints, higher baseline fertilizer villages may be the richest and therefore most able to respond to the shock.

Left unanswered so far is which of these two broad sources drive baseline fertilizer use. The reality of production, especially in agriculture, is a combination of the two. Both are visible in our data. First, there is a strong correlation between fertilizer use and finance across villages in Rwanda. We use our baseline data across all villages to measure debt by village. Figure 3a plots a bin scatter of fertilizer expenditures as a share of harvest and its relationship to village debt. Figure 3b shows that higher debt correlates with walking distance to banks.

Figure 3: Fertilizer and External Finance

(a) External Debt and Fertilizer's Share of Harvest

(b) Average Walking Time to Bank

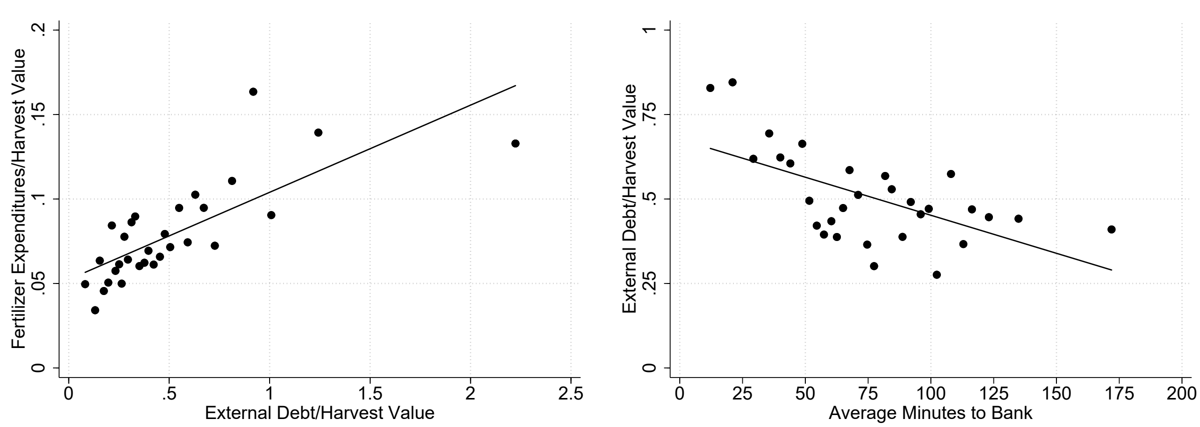

Figure Notes: Panel (a) plots fertilizer expenditures as a share of harvest value for all 444 villages and its relationship to total external debt held by the village. Panel (b) relates external finance levels to average walking time to the bank. Both sub-figures use bin scatters with 30 bins.

Figure 4: Official Government Fertilizer Recommendation Zones for Potatoes

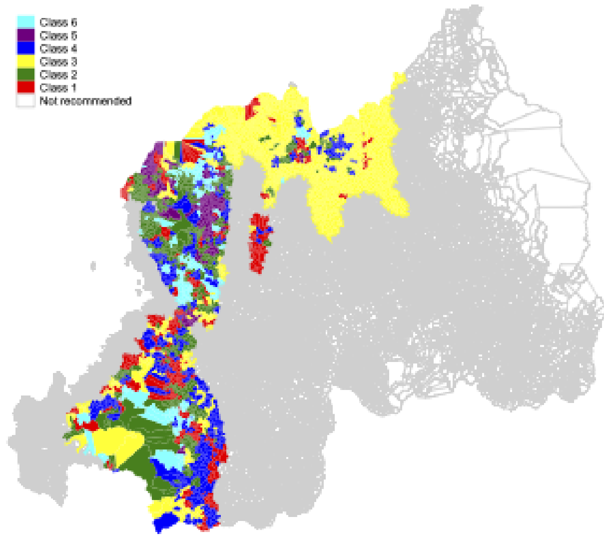

Figure Notes: This figure plots the 6 official fertilizer zone recommendations based on underlying soil characteristics and the combinations of DAP, NPK, and urea that are recommended by village. Roughly, the higher the class, the more intensive fertilizer use is required on the land. "N.R." stands for "not recommended."

On the other hand, technology also varies systematically even within small geographic areas. Rwanda is a hilly country (especially in the west), which implies substantial variation in soil quality – erosion, carbon, and nitrogen levels – that are critical inputs to the yield response to fertilizer.

Starting in 2018, the Rwandan government conducted thousands of soil samples around the country to build plot-level fertilizer recommendations for farmers. The Rwandan Soil Information System (RwaSIS) allows farmers a web portal to input their unique land identifier (UPI) and receive crop-specific fertilizer recommendations. We scrape this data from the website and plot the government-recommended fertilizer levels for potatoes grown in western Rwanda. Roughly, higher classes represent more intensive fertilizer per hectare recommendations.[16] Figure 4 shows that government-recommended fertilizer intensity varies even within relatively small geographic units, implying substantial heterogeneity in the technological returns to fertilizer across villages (see also Marenya and Barrett, 2009; Theriault et al., 2018, in Kenya and Burkina Faso on this point).

Household-Level Evidence These same patterns hold at the household level. We regress household-level fertilizer expenditures per hectare in 2020 on detailed soil characteristics (pH, nitrogen, organic carbon, and calcium) and bank distance. These soil characteristics

[16] Since potatoes use primarily the balanced NPK 17-17-17 fertilizer blend, higher classes roughly represent more NPK and urea topdressing. The dataset itself comes from the Rwandan Soil Information System (RwaSIS). This online system offers farmers specific recommendations that they can access by inputting their unique land identifier (UPI), constructed in 2021 after 4 years of soil sampling across the country. We scrape this data from the website.

are derived from the same detailed soil tests used as inputs into the optimal fertilizer recommendations of Figure 4, but have the benefit of being country-wide. We scrape these data from the web and link them to households in our data via GPS coordinates. We provide standard errors clustered at the village and the district level to account for potential spatial correlation in soil types.

Table 3: Correlates of Fertilizer per Hectare use at the Household Level in 2020

	(1)
Log Minutes to Bank	-0.069
	(0.031)**
	[0.058]
Carbon	
Slightly Low (2.51-3)	0.169***
	(0.050)***
	[0.087]*
Optimal (> 3)	0.659
	(0.085)***
	[0.145]***
Nitrogen	
Slightly Low (0.2-0.5)	-0.158
	(0.062)**
	[0.104]
pH Level	
Low (5.01-5.5)	-0.227
	(0.067)***
	[0.133]
Slightly Low (5.51-6.0)	-0.790
	(0.115)***
	[0.231]***
Optimal (6.01-7.0)	-1.189
	(0.128)***
	[0.254]***
Calcium	
Low (500.01-1000)	0.094
	(0.062)
	[0.095]
Slightly Low (1000.01-1600)	0.768
	(0.115)***
	[0.251]***
Optimal (1600.01-2400)	1.415
	(0.105)***
	[0.135]***
High (>2400)	1.637
	(0.129)***
	[0.227]***
Obs	8,581
R2	0.073

Table notes: Standard errors clustered by village and district are in parentheses and brackets. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***. Unlisted bins imply that there is no soil realization of that type. For example, all soil in Rwanda has nitrogen levels ≤ 0.5, so the table includes only the base "low" and the reported "slightly low" bins.

Table 3 shows two main results. First, bank distance is negatively correlated with fertilizer use at the household level, as was true with the village-level patterns. Second, soil characteristics are also highly correlated with fertilizer expenditures along several dimensions. Better nitrogen levels are associated with less need for fertilizer.[17] Higher organic carbon – a key input into the yield response of fertilizer – is positively correlated with expenditures. These results highlight that our notion of technological suitability is not just capturing high-

[17] All Rwandan soil is low in nitrogen, and all cells are either "low" or "slightly" low.

versus low-productivity land. Soils with balanced pH levels and high nitrogen soils are both high quality, yet require less fertilizer to optimize.[18]

Together, these results characterize the reality of production in this environment: distorted owner-operators make decisions in the presence of competitors with potentially substitutable products, but different input mixes due to technology differences. Understanding the interplay between technology and frictions will matter for policy design. We next build a model to accomplish this task.

4 Model

The model is cast in discrete time where a time period is a year. We model Rwanda as a small open economy in which fertilizer is imported at the exogenous global price p_{xt}^{market}, consistent with what we observe empirically.

Sectors and Village Structure There are two sectors: non-agriculture, or "manufacturing" m for short, and agriculture a. Within the economy are a continuum of villages, denoted $j \in \mathcal{J}$. Each village contains a unit measure of *ex ante* identical households that live forever, but will be heterogeneous *ex post* due to idiosyncratic productivity shocks and the corresponding savings decisions. Each household derives utility from each of the two sectors' final goods, and utility flow is

$$\sum_{t=0}^{\infty} \beta^t \frac{\left((c_{at} - \overline{a})^{\zeta} c_{mt}^{1-\zeta}\right)^{1-\chi} - 1}{1-\chi},$$

where $\overline{a}$ is a subsistence requirement in agriculture and β the discount factor. Households save s denominated in the agricultural final good with exogenous gross return R (which may be less than one due to storage costs or depreciation) and are restricted to savings, $s \geq 0$.

Villages differ in two ways: their technological fertilizer intensity α_j and the severity of their working capital constraint ϕ_j, both of which we will formalize below. The c.d.f. $G(\alpha, \phi)$ summarizes the distribution of villages across these two characteristics. To ease some notation, we will integrate over villages j below, but that should be understood to be an integration with weights given by the distribution G.

Manufacturing Production The manufacturing sector produces its final consumption good according to the production function $Y_{mt} = AH_{mt}$ where H_{mt} is efficiency units of labor hired in a centralized, competitive market that draws from all villages. We normalize the

[18] Given these results, we could have alternatively instrumented baseline fertilizer intensity with soil characteristics for our empirical results. Our model provides an explicit path by which this violates the exclusion restriction. Soil characteristics impact income, which in turn impacts the tightness of financial constraints via savings.

output price of the manufacturing sector to $p_m = 1$. In equilibrium, the wage per efficiency unit is $w_{mt} = A$ for all t.

Agricultural Production Each household i in village j produces a village-specific crop. They can use one of two technologies available to produce it. The modern production function is of the form,

$$y^M_{ijt} = z_{ijt}\left(\alpha_j^{\frac{1}{\sigma}} x_{ijt}^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)^{\frac{1}{\sigma}} n_{ijt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\gamma\sigma}{\sigma-1}}$$

where input choices are x is fertilizer and n is labor. α_j is a feature of the technology and sub-scripted, meaning that it differs across villages. All villages have the same returns to scale $\gamma \in (0,1)$. Input choices are made after the realization of z, a household-level productivity shock. We adopt a similar process to Buera et al. (2011), where each period with probability $1-\psi$ a household receives a new draw $z_{i,t+1} \sim \text{Pareto}(\underline{z}_M, \theta_M)$ and with probability ψ, $z_{i,t+1} = z_{i,t}$. We summarize this c.d.f. as $Q(z_{t+1}, z_t)$. The transition matrix is identical across villages.

A household can alternatively choose to use the traditional technology,

$$y^T_{ijt} = A_T n^\gamma_{ijt}.$$

The traditional technology uses only labor and no skill, and has productivity shifter A_T that is constant over time.[19]

Regardless of technology, input choices are constrained by a working capital constraint that varies across villages,

$$p_x x + w_{aj} n \leq \phi_j s$$

where p_x is the price of fertilizer and w_{aj} is the village-specific equilibrium wage per unit of labor. This constraint says that household i in village j can leverage savings s to pay for inputs up to a multiple ϕ_j. A household therefore chooses the technology that maximizes profit subject to the working capital constraint

$$\begin{aligned}
\pi(s,z) &= \max\{\pi^T(s), \pi^M(s,z)\} \\
\text{subject to:} \quad \pi^T(s) &= \max_n \; p_{aj} A_T n^\gamma - w_{aj} n \\
\pi^M(s,z) &= \max_{n,x} \; p_{aj} z \left(\alpha_j^{\frac{1}{\sigma}} x^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)^{\frac{1}{\sigma}} n^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\gamma\sigma}{\sigma-1}} - w_{aj} n - p_x x \\
p_x x + w_{aj} n &\leq \phi_j s
\end{aligned}$$

[19] We use "modern" and "traditional" technologies here as short-hand to distinguish only the use of fertilizer. There is almost no tractor use or the large-scale farming that would take advantage of the increasing returns exploited in richer countries.

where p_{aj} is the relevant output price in village j. Overall, a village is defined by its technological fertilizer intensity α_j and the tightness of its working capital constraint ϕ_j.

Agricultural Consumption Good The village-specific crops are transformed into the final agricultural consumption good by a CES aggregator

$$Y_a = \left(\int_j y_j^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$$

so that a stand-in firm has the profit function

$$\max_{y_{aj}} \; p_{ac} \left(\int_j y_j^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} - \int_j p_{aj} y_{aj} dj.$$

Here, p_{ac} is the price of the final agricultural consumption good paid by consumers and p_{aj} is the output price received by households in village j for their production. The continuum of households within any village implies no household market power for the differentiated products.

Household Labor Allocation Each worker in village j can work inside the village at the local market clearing wage w_{aj} or in the centralized manufacturing market for wage w_m. Individuals are identical in agriculture but receive draw ξ_m in manufacturing. Each individual worker earns

$$\text{Earnings}(\xi_m) = \max\{w_a, \xi_m w_m\}.$$

We assume throughout that ξ_m is distributed Pareto with minimum value 1 and shape θ. Because there are a continuum of individuals within a household, total household i labor earnings as

$$\mathcal{W}_{ijt} = w_{ajt} + \frac{w_{ajt}^{1-\theta} w_{mt}^{\theta}}{\theta - 1} \tag{4.1}$$

This is identical for all households in village j, so we drop the i subscript and write $\mathcal{W}_{jt}$ going forward.

Fertilizer, Subsidies, and Government Budgets Fertilizer is imported and the exogenous global market price of fertilizer is p_{xt}^{market}.

The government has access to two policy instruments. The first is a fertilizer subsidy τ_{xt} that is identical in all villages. Farmers pay $p_{xt} := (1 - \tau_{xt}) p_{xt}^{\text{market}}$ per unit of fertilizer and the government incurs a cost of $\tau_{xt} p_{xt}^{\text{market}}$ on each unit. That subsidy is financed by taxes, subject to a balanced budget constraint, where government revenue is collected via a VAT tax on non-agricultural consumption.[20] The government earns τ_{mt} on each unit given the normalized manufacturing output price.

The second instrument is a set of village-specific *ex post* transfers T_{jt}, such that $\int_j T_{jt} dj = 0$ for all periods. They are *ex post* in the sense that they cannot fund production in the current period but can add to, or subtract from, disposable income. While these are not prevalent in the context of Rwandan policy, we will show below that they are useful to disentangle the planner's production-side concerns with her interest in redistributing consumption.

Household Problem in Village j The timing of the model works as follows (dropping the (i, j) subscripts for simplicity). At $t-1$, households choose savings s_t to bring into period t. Then the shock z_t is realized. Households then make decisions on farm inputs and the allocation of labor, income is realized, and consumption and savings decisions are made.

The individual state of the household is therefore (z, s), and the aggregate state is the distribution μ across individual states and villages along with the fertilizer price and tax system $\boldsymbol{\tau}_t := (p_{xt}^{\text{market}}, \tau_{xt}, \tau_{mt}, T_{jt})$. The household knows the full sequence $\{\boldsymbol{\tau}_t\}_{t=0}^{\infty}$ when making decisions. Recursively, the problem is

$$
\begin{aligned}
v_j(z, s, \mu, \boldsymbol{\tau}) \; &= \max_{c_a, c_m, s', \phi \in \{0,1\}} \frac{\left((c_{at} - \overline{a})^{\zeta} c_{mt}^{1-\zeta}\right)^{1-\chi} - 1}{1-\chi} + \beta \int_{z'} v_j(z', s', \mu', \boldsymbol{\tau}') dQ(z', z) \\
s.t. \quad & p_{ac} c_a + (1+\tau_m) c_m + p_{ac}(s' - Rs) = \max\{\pi^M, \pi^T\} + \mathcal{W}_j + T_j \\
& \pi^M = \max_{x,n} \; p_{aj} z \left(\alpha_j^{\frac{1}{\sigma}} x^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)^{\frac{1}{\sigma}} n^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\gamma\sigma}{\sigma-1}} - w_{aj} n - (1-\tau_x) p_x^{\text{market}} x \\
& \pi^T = \max_{n} \; p_{aj} A_T n^{\gamma} - w_{aj} n \\
& (1-\tau_x) p_x^{\text{market}} x + w_{aj} n \leq \phi_j s \\
& s' \geq 0
\end{aligned}
$$

The first constraint is the budget constraint, the next two define profit for both technologies, the fourth is the financial friction, and the fifth is the no-borrowing constraint. We define the decision rule $\phi(s, z, \mu, \boldsymbol{\tau}) = 1$ if the household chooses the modern farming technology.

We note here that when we introduce a shock to the fertilizer price p_x^{market}, we start the economy at its stationary equilibrium with baseline price p_{x0}^{market} (motivated by the stability

[20] In Rwanda, expenditures on agricultural inputs and outputs are exempt from VAT. And while there is in principle income taxes on agricultural profits, the thresholds exempt nearly all non-commercial-scale farming. Farm incomes must be above 12 million RWF (9,200 USD in 2024, about 9 times GDP per capita) to incur any tax burden. Other taxes are designed to capture revenue from the agricultural sector, but play a minor role. For example, market fees are paid by traders who operate stalls in markets, but these are small revenue generators and we exclude them for simplicity. Similarly, non-agricultural earnings are subject to income taxes (above a threshold), but given that much of this work is informal in our setting, we refrain from modeling this tax on non-agricultural wages.

of prices and subsidies in Section 2), then unexpectedly shock it with a deterministic sequence of prices $\{p_{xt}^{\text{market}}\}_{t=1}^{t=\infty}$. Hence, the recursive program above includes perfect foresight of the evolution of the aggregate state μ.

4.1 Recursive Competitive Equilibrium

Given a sequence $\{\tau_{xt}, T_{jt}, p_{xt}^{\text{market}}\}$, a recursive competitive equilibrium of this model is a set of decision rules for households $(c_a, c_{mj}, x_j^M, n_{aj}^M, n_{aj}^T, s_j', \psi)$, the manufacturing firm N_m, and the final agricultural good producer y_{aj}, prices $\{p_{aj}\}, p_{ac}, w_a$ and tax rate τ_m such that (i) the household's decision rules are consistent with its optimization problem given prices and taxes, (ii) N_m solves the manufacturing firm's decision problem, (iii), y_{aj} solves the agricultural final goods firm decision problem, (iv) the law of motion for the aggregate state, $\mu' := \Lambda(\mu)$, is consistent with the decision rules, and (v) the government balances its budget and markets clear:

1. Government budget balance:

$$\text{Subsidy Budget:} \qquad \tau_x p_x^{market} \int_j \int_{s,z} x_j(s,z) d\mu_j dj = \tau_m \int_j \int_{s,z} c_{mj}(s,z) d\mu_j dj$$

$$\text{Net-Zero Transfers:} \qquad \int_j T_j dj = 0$$

2. Market clearing:

   (a) Agricultural market: for each j,

$$\begin{aligned} y_{aj} &= \int_{s,z} \psi(s,z) z \left(\alpha_j^{\frac{1}{\sigma}} x_j(s,z)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)^{\frac{1}{\sigma}} n_j(s,z)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\gamma\sigma}{\sigma-1}} \mu_j + \\ &+ \int_{s,z} (1-\psi(s,z)) A_T n_j^T(s,z)^{\gamma} d\mu_j \end{aligned}$$

   (b) Agricultural final goods market:

$$\left(\int_j y_{aj}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} = \int_j \int_{(s,z)} c_{aj}(s,z) d\mu_j dj$$

   (c) The agricultural labor market: for each j,

$$\int_{s,z} n_j(s,z) d\mu_j = 1 - \left(\frac{w_m}{w_{aj}} \right)^{\theta}$$

(d) The manufacturing labor market:

$$H_m = \frac{\theta}{\theta - 1} \int_j \left(\frac{w_m}{w_{aj}} \right)^{\theta-1} dj$$

A stationary equilibrium involves a constant sequence for subsidies, transfers, and fertilizer market prices, $\{\tau_{xt}, T_{jt}, p_{xt}^{market}\} = (\tau_{x,ss}, T_{j,ss}, p_{x,ss}^{\text{market}})$ for all t and an aggregate state μ^* such that $\mu^* = \Lambda(\mu^*)$.

4.1.1 Ramsey Planner

The recursive competitive equilibrium above takes as given $\{\tau_{xt}, T_{jt}, p_{xt}^{\text{market}}\}$. The planner then chooses a sequence of subsidies $\{\tau_{xt}, T_{jt}\}$ given the market price sequence $\{p_{xt}^{\text{market}}\}$. We assume she cannot commit to future policy.[21] Therefore, our notion of equilibrium is a Markov-perfect equilibrium, in which the planner has no profitable deviations available. The planner maximizes

$$W(\mu_0, \{\tau_{xt}, T_{jt}\}) = \int_{z,s,j} v_j(z, s, \mu_0, \boldsymbol{\tau}) d\mu_0(z, s, j)$$

subject to the resulting recursive competitive equilibrium and the condition that there are no welfare-improving deviations from $\{\tau_{xt}, T_{jt}\}$ at any period t. A stationary Markov equilibrium is a recursive competitive equilibrium such that when $\mu_0 = \mu^*$ and $p_{xt}^{\text{market}} = p_{x,ss}^{\text{market}}$ for all t, the planner chooses a value τ_x^* such that $(\tau_x^*, T_j^*) = \text{argmax } W(\mu^*, \tau_x^*, T_j^*)$ subject to a similar no deviation condition.

5 Analytic Results

Our model includes several features that may seem extraneous at first glance. Several are important quantitatively, but do not necessarily affect the qualitative trade-off that is central to the model. To highlight this channel more directly, we strip the model of its extraneous features and analytically characterize a simpler version that will look familiar to readers of standard macro-development models with an agricultural sector, such as Restuccia et al. (2008).

To that end, we make the model static, which requires a small change in the financial friction to remove savings, $p_x x + w_a n \leq \phi_j$ where now ϕ_j is an exogenous parameter in village

[21] Policy with commitment is not time consistent in this model. Therefore, any exercise in which we start the economy from its stationary equilibrium, then allow the planner to adjust policy, will lead to optimal subsidy adjustments even in the absence of price changes. That is, our quantitative exercise both changes the price and breaks commitment. In addition to its realism, assuming no commitment implies that the planner would never adjust the subsidy in the absence of a price change. We discuss this more in the quantitative results.

j instead of a multiple of savings. It also makes any notion of planner commitment irrelevant. We make two additional assumptions. First, we assume a Cobb-Douglas production function for the modern sector and remove the traditional sector. Village j therefore has the production function $y_j = x^{\alpha_j} n^{\eta_j}$, where $\eta_j = \gamma - \alpha_j$. We will show in the quantitative results that our data point us toward Cobb-Douglas production. We remove idiosyncratic productivity as well, but this is purely notational and has no bearing on the results here. Second, we assume away skills in non-agricultural labor, so that each individual chooses $\max\{w_{aj}, w_m\}$ leading to the result that $w_{aj} = w_m = A$ for all villages j.[22]

In this simplified model, a household in village j solves

$$\begin{aligned} &\max_{c_a, c_m, x, n_a} \quad \zeta \log(c_a) + (1-\zeta)\log(c_m) \\ s.t. \quad & p_{ac} c_a + (1+\tau_m) c_m = p_{aj} x^{\alpha_j} n^{\eta_j} - (1-\tau_x) p_x^{\text{market}} x - wn + w + T_j \\ & (1-\tau_x) p_x^{\text{market}} x + wn \leq \phi_j \end{aligned}$$

We define the competitive equilibrium of this simplified economy in Appendix B, though it differs only slightly from the one defined above.

Appendix D includes all proofs, along with the details of derivations skipped in the main text here.

5.1 Characterizing the Frictionless Equilibrium

To understand policy should respond to fertilizer price changes, it is helpful to start from the frictionless economy with $\phi_j = \infty$ for all villages. Characterizing this economy allows us to show how equilibrium forces work to reallocate production across heterogeneous villages. We then use this as a benchmark for the economy with binding financial frictions.

Because we are interested in changes between two equilibria, it turns out to be much simpler to characterize outcomes in differences instead of levels. To start, Lemma 1 characterizes the new equilibrium (with higher p_x^{market}) in terms of the baseline.

Lemma 1. *Assume that the market fertilizer price rises to $p_{x2}^{market} > p_{x1}^{market}$. Define*

$$\mathcal{Y} = \frac{\int_j y_{j1}^{\frac{\nu-1}{\nu}} dj}{\int_j \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{j1}^{\frac{\nu-1}{\nu}} dj}.$$

This value $\mathcal{Y}$, combined with exogenous parameters, fully characterizes the p_{x2}^{market} frictionless

[22] A common way to break this equivalence is to add an exogenous tax on non-agricultural wages, so that $w_a = (1-\tau)w_m$ (e.g. Restuccia et al., 2008). Adding such a term does not affect our results in this section.

equilibrium relative to the p_{x1}^{market} *equilibrium. For example, the equilibrium price ratios are*

$$\frac{p_{ac2}}{p_{ac1}} = \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma}{\nu-1}} \qquad \textit{and} \qquad \frac{p_{aj2}}{p_{aj1}} = \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma-1}{\nu-1}} \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j}{\nu-(\nu-1)\gamma}}$$

The term $\mathcal{Y}$ determines the ratio of baseline aggregate output to an adjusted version of itself, where the adjustment depends on the change to the fertilizer price p_x.[23] Thus, it summarizes the aggregate decline in production from the higher fertilizer price. The key feature of $\mathcal{Y}$ is that it depends on no endogenous values from the new p_{x2} equilibrium. This fully characterizes the new equilibrium in terms of the baseline one, which lets us side-step many of the issues associated with working in levels. Instead we characterize equilibrium changes in differences. To that end, we will throughout define $\widehat{y}$ as the log difference in some variable y between the p_{x2}^{market} and p_{x1}^{market} equilibria.

We summarize the economy by its CES Domar weights $\Gamma_j = (y_j/Y_a)^{\frac{\nu-1}{\nu}}$. Γ_j measures the share of production that takes place in village j. Combining Lemma 1 with a bit of algebra gives us a simple form of their evolution across villages after a fertilizer price shock,

$$\frac{\partial^2 \widehat{\Gamma}_j}{\partial \widehat{p}_x^{\text{market}} \partial \alpha_j} = \frac{1-\nu}{\nu-(\nu-1)\gamma} \begin{cases} < 0, & \text{if } \nu > 1 \\ = 0, & \text{if } \nu = 1 \\ > 0, & \text{if } \nu < 1 \end{cases} \tag{5.1}$$

(5.1) characterizes the distributional consequences for both production and income of the fertilizer price shock in an economy with unconstrained production decisions, since farm profit shares the same derivative $\frac{\partial^2 \widehat{\pi}_j}{\partial \widehat{p}_x^{\text{market}} \partial \alpha_j} = \frac{\partial^2 \widehat{\Gamma}_j}{\partial \widehat{p}_x^{\text{market}} \partial \alpha_j}$.[24]

If villages produce substitutes ($\nu > 1$), the economy can easily reallocate production toward relatively less exposed low-α villages. In this case, equilibrium price adjustments shift production to low α villages. That is, (5.1) is negative. When villages are complements ($\nu < 1$), the scope for inter-village substitution is limited. Instead, prices push production the other way: they help cushion high-α villages, consequently reversing the cross-sectional slope of share response in (5.1).

This offers a baseline of how the economy should evolve after a fertilizer price shock. Next, we show how financial frictions impede this equilibrium reallocation and offer a productive efficiency rationale for subsidies.

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[23] In the proof of Lemma 1, we show that $Y_{a2}/Y_{a1} = 1/\mathcal{Y}^{\frac{\nu}{\nu-1}-\gamma}$.

[24] This follows from noting that

$$\pi_{j2} = \underbrace{\mathcal{Y}\left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}}}_{\equiv \text{ adjustment to } \pi_{j1}} \underbrace{(1-\gamma)p_{j1}y_{j1}}_{\equiv \pi_{j1}}.$$

The relevant derivations are in the proof of Lemma 1.

5.2 Misallocation and Optimal Policy with Financial Frictions

We now return to the frictional equilibrium, where each village is subject to $p_x x + wn \leq \phi_j$. The baseline economy is characterized by fertilizer state $(p_{x1}^{\text{market}}, \tau_{x1})$, implying farmer price $p_{x1} = (1 - \tau_{x1})p_{x1}^{\text{market}}$. To make the results as sharp as possible, we assume that ϕ_j is tight enough that it binds for all j at baseline farmer price p_{x1} (which also guarantees it binds at $p_{x2}^{\text{market}} > p_{x1}^{\text{market}}$ for a fixed subsidy level). Thus, we study an economy in which financial frictions are extremely tight, but still heterogeneous across villages. Input choices are

$$x_j^c = \frac{\alpha_j \phi_j}{\gamma p_x} \qquad \text{and} \qquad n_j^c = \frac{\eta_j \phi_j}{\gamma w} \tag{5.2}$$

where the superscript c is a reminder that these come from the "constrained" frictional equilibrium (and we will sometimes use superscript u to denote the frictionless or "unconstrained" economy).

Notice that the input choices do not depend on output prices, which is a natural consequence of binding financial constraints. Intuitively, if the output price rises, but costs cannot, there is little a farmer can do to adjust. But these price adjustments are the driving force behind frictionless reallocation. We formalize the consequences of this lower output price elasticity with a similar approach to the unconstrained case. We show in Appendix D that the same approach used in Lemma 1 works here, which, as before, lets us characterize the evolution of the Domar weights under binding financial frictions,

$$\frac{\partial^2 \widehat{\Gamma}_j^c}{\partial \widehat{p}_x^{\text{market}} \partial \alpha_j} = \frac{1-\nu}{\nu} \begin{cases} < 0, & \text{if } \nu > 1 \\ = 0, & \text{if } \nu = 1 \\ > 0, & \text{if } \nu < 1 \end{cases} \tag{5.3}$$

The basic intuition stays the same: if villages are substitutes, production shifts toward less exposed low-α villages, and the opposite is true if complements. But importantly, financial frictions change the *magnitude* because the denominator is missing the $-(\nu - 1)\gamma$ term that appears in the unconstrained case in (5.1). This is the formalization of financial frictions. They decrease the output price elasticity of input choices and thereby mute equilibrium price effects. As a consequence, the frictional economy cannot deliver the reallocation demanded by the frictionless equilibrium. This pattern holds true for any distribution of financial frictions and technology, as the long as the marginal distribution of technology is not degenerate.

For instance, when $\nu > 1$, both cross-partials are negative but

$$\left|\frac{\partial^2 \widehat{\Gamma}^u_j}{\partial \widehat{p}^{\text{market}}_x \partial \alpha_j}\right| > \left|\frac{\partial^2 \widehat{\Gamma}^c_j}{\partial \widehat{p}^{\text{market}}_x \partial \alpha_j}\right|.$$

That is, in the $\nu > 1$ case where reallocation is warranted, the frictional equilibrium delivers less of it. On the other hand, when $\nu \in (0, 1)$, the unconstrained economy demands relatively similar Domar weights across villages. To achieve this, GE price adjustments dampen the cross-sectional sensitivity to fertilizer price changes. With financial frictions, that dampening cannot occur, and the frictional equilibrium instead *over*-adjusts the shares.

A perhaps simpler way to see this is to ask how fertilizer in the frictional economy evolves relative to the frictionless one for the same shock. Specifically, we can measure the ratio of village j's fertilizer use from its frictionless benchmark as $\kappa^x_j := x^u_j / x^c_j$, inclusive of price differences between the two equilibria.[25] Using Lemma 1 to characterize equilibrium price changes in the frictionless equilibrium allows a "diff-in-diff" characterization of how the fertilizer gap evolves between villages j and k,

$$\widehat{\kappa}^x_j - \widehat{\kappa}^x_k = (\alpha_k - \alpha_j)\left(\frac{\nu - 1}{\nu - (\nu - 1)\gamma}\right)\widehat{p}_x. \tag{5.4}$$

(5.4) summarizes the discussion above: no matter where the frictionless economy wants to induce production, the frictional economy always fails to deliver it. It keeps too much fertilizer in high-α villages when $\nu > 1$ and too little when $\nu < 1$.[26] This lack of productive reallocation manifests in a simple summary of how village-level distortions change, which we summarize in Proposition 1.

Proposition 1. *Assume the market fertilizer price rises from p^{market}_{x1} to p^{market}_{x2} for a fixed policy $(\tau_x, \{T_j\})$. Define the shadow value of fertilizer in village j as*

$$1 + \lambda^c_{jt} = \frac{\alpha_j p_{aj} x^{\alpha_j - 1} n^{\eta_j}}{(1 - \tau_x) p^{market}_{xt}}.$$

[25] This is equivalent to discussing in terms of output, because $\Delta_t \kappa^x_j = (1/\gamma)\Delta_t[\log(y^u_j) - \log(y^c_j)]$.

[26] To see this more clearly, re-arrange κ^x_j. For two villages with $\alpha_H > \alpha_L$, writing (5.4) in this manner implies

$$\begin{aligned} \widehat{x}^c_H - \widehat{x}^c_L &> \widehat{x}^u_H - \widehat{x}^u_L \quad \text{if } \nu > 1 \\ \widehat{x}^c_H - \widehat{x}^c_L &= \widehat{x}^u_H - \widehat{x}^u_L \quad \text{if } \nu = 1 \\ \widehat{x}^c_H - \widehat{x}^c_L &< \widehat{x}^u_H - \widehat{x}^u_L \quad \text{if } \nu < 1. \end{aligned}$$

The left-hand side measures the relative fertilizer changes between H and L in the frictional equilibrium and compares them to their frictionless counterparts on the right, which can be characterized by applying Lemma 1.

The equilibrium fertilizer shadow prices $\{\lambda_{jt}^c\}$ *evolve across villages as*

$$\frac{\partial^2 \widehat{1+\lambda_j^c}}{\partial \widehat{p}_x^{market} \partial \alpha_j} = \frac{\partial^2 \widehat{\Gamma}_j^c}{\partial \widehat{p}_x^{market} \partial \alpha_j} = \frac{1-\nu}{\nu}$$

for any non-degenerate distribution $G(\alpha, \phi)$.

Proposition 1 characterizes the evolution of the Restuccia and Rogerson (2008)-style distortion $1+\lambda_j^c$ across villages. They rise most in low-α villages when villages are substitutes ($\nu > 1$) and most in high-α villages if complements. That is, the distortion rises in exactly the villages toward which the efficient economy shifts production. This opens scope for a subsidy to deliver it instead, leading to a simple characterization of the qualitative properties of the optimal subsidy.

Proposition 2. *If the planner maximizes utilitarian welfare with her full set of instruments* $(\tau_x, \{T_j\})$,

$$\max_{\tau_x, \{T_j\}} \quad \int_j \Big[\zeta \log(c_{aj}) + (1-\zeta)\log(c_{mj})\Big] dj$$

subject to consistency with the competitive equilibrium, then for any non-degenerate distribution $G(\alpha, \phi)$,

$$\frac{\partial \tau_x^*}{\partial p_x^{market}} \begin{cases} > 0, & \text{if } \nu < 1 \\ = 0, & \text{if } \nu = 1 \\ < 0, & \text{if } \nu > 1 \end{cases}$$

If the marginal distribution of α *is degenerate, then* $\frac{\partial \tau_x^*}{\partial p_x^{market}} = 0$ *in all cases.*

This characterization of optimal policy flows closely the discussion above. If villages are substitutes, the planner tries to shift production shares toward low-α villages, which she does by lowering the subsidy. If complements, she cushions high-α villages by increasing the subsidy. The key sign restriction we derive in the proof is

$$\text{sign}\left(\frac{\partial \tau_x^*}{\partial p_x^{\text{market}}}\right) = -\text{sign}\left(\frac{\nu - 1}{\nu} \text{Var}_\Gamma[\alpha]\right) \tag{5.5}$$

where Var_Γ is variance of technology α under probability distribution defined by the Domar weights Γ_j^c.

One of the interesting features of this section is that we have not specified much about the distribution of financial frictions or technology, other than to require that the marginal technology distribution is non-degenerate (and that financial frictions bind). (5.5) shows why. Village-level variation in financial frictions affects only the underlying Domar weights

Γ. The key role for policy here is to shift production to villages with different technology. Indeed, if $\text{Var}_\Gamma[\alpha] = 0$ then optimal subsidy does not vary with the price regardless of financial frictions. Financial frictions are captured in the baseline level of the subsidy.

These results point to two important lessons for quantifying the model. First, mis-attributing variation in fertilizer use to financial frictions – for example, by assuming identical production technologies across villages – will cause us to under-value the importance of this productive reallocation channel and change the implications for optimal policy. Second, the ease of substitutability across village production will play an important role both in the sign of policy adjustments and the quantitative magnitude through ν.

5.3 Inequality with Missing Instruments and Mapping to the Data

Proposition 2 is stark, in the sense that it emphasizes exclusively the role of technology in optimal policy. This is because the planner has sufficient redistributional instruments to decouple production and consumption. In reality, these village-specific *ex post* transfers may be difficult to execute in weak institutional environments. We do not see transfers responding to the shock in Rwanda. We next turn to characterizing the economy in which the planner is required to set transfers $T_j = 0$ for all villages j. In this case, the subsidy balances the productive reallocation described above with issues related to inequality. Lemma 2 shows we can separate these two forces in the welfare function and thus characterize them independently.

Lemma 2. *If the planner maximizes utilitarian welfare with access to ex post transfers, the optimal choice of subsidy solves*

$$\max_{\tau_x} \quad \zeta \log(Y_a) - (1-\zeta)\log(1+\tau_m). \tag{5.6}$$

and, if $T_j = 0$ for all j,

$$\max_{\tau_x} \quad \zeta \log(Y_a) + \int_j \log(C_j)dj - (1-\zeta)\log(1+\tau_m) \tag{5.7}$$

where $C_j = \frac{\zeta}{1-\zeta}(w - \mathbb{E}[\phi])\Gamma_j - \phi_j + w$ is total equilibrium consumption expenditures in village j.

The inability to redistribute *ex post* creates the additional term in (5.7) that takes into account consumption redistribution separate from production. Holding policy fixed, a shock to p_x^{market} affects aggregate welfare by

$$\frac{\partial W}{\partial \log(p_x^{\text{market}})} = \zeta \mathbb{E}_\Gamma[-\alpha] - \frac{\nu-1}{\nu}\zeta \overline{C} Cov_\Gamma\left(\frac{1}{C_j}, \alpha_j\right) \tag{5.8}$$

where $\overline{C} := \int_j C_j dj = (w - \mathbb{E}[\phi])/(1-\zeta)$ is total consumption expenditures in the economy. The first term is output exposure. Per Lemma 2, this is identical to the case with *ex post* transfers. $\partial \log(y_j)/\partial \log(p_x^{\text{market}}) = -\alpha_j$, which is then aggregated by the Domar weights measuring j's importance to the aggregate. The second term is entirely about consumption redistribution.[27] The covariance term measures which villages have higher marginal utility, then ν governs whether the distribution of production and income pushes resources toward or away from them.

The policy question then becomes whether adjusting the net fertilizer price $p_x = (1-\tau_x)p_x^{\text{market}}$ counteracts the welfare consequences of the shock or reinforces it. The earlier results cover the first term: policy should direct the net price even higher when $\nu > 1$ (i.e., τ_x falls) and lower when $\nu < 1$.[28] Now, it also depends on how that production force affects the covariance between marginal utility and technology. Working through the algebra of this problem yields an adjusted sign restriction

$$\text{sign}\left(\frac{\partial \tau_x^*}{\partial p_x^{\text{market}}}\right) = -\text{sign}\left(\frac{\nu-1}{\nu}\text{Var}_\Gamma[\alpha]\right)\text{sign}(\mathcal{B}) \tag{5.9}$$

where

$$\mathcal{B} = 1 - \overline{C}\left(\frac{\partial \text{Cov}_\Gamma\left(\frac{1}{C_j}, \alpha_j\right)/\partial \log(p_x)}{\text{Var}_\Gamma[\alpha]}\right)$$

Notice that if covariance is zero, as is this case with *ex post* transfers, this collapses to the previous case. The simplicity of the previous result is eliminated here by non-trivial covariance in $\mathcal{B}$. But (5.9) offers something almost as helpful: it points to a key moment in this economy. Specifically, take the Domar-weighted OLS regression of the form

$$MU(c_{hjt}) = \eta(Post_t \times X_{j0}) + \theta_j + \beta_t + \varepsilon_{hjt}$$

for household h in village j at time t, where X_{j0} is total village baseline fertilizer expenditure and $MU(c_{hjt})$ is marginal utility (here $= 1/c_{hjt}$). This looks strikingly similar to the

[27] While at first glance it looks like this term involves a level effect, the CES properties of the model imply that it does not. This follows from noting that

$$\int_j \frac{\partial C_j}{\partial \log(p_x^{\text{market}})} = \zeta\overline{C}\int_j \frac{\partial \Gamma_j}{\partial \log(p_x^{\text{market}})} = 0$$

since $\int_j \Gamma_j dj = 1$. Thus, there is no aggregate change in consumption, only redistribution.

[28] Explicitly, the derivative of that term is $\left(\frac{\nu-1}{\nu}\right)\text{Var}_\Gamma[\alpha]$ as is required to sign the case with *ex post* transfers.

regressions we ran in Section 3 with two adjustments. The first is that it is weighted by the Domar weights, so that averaging takes into account where production actually occurs. The second is that it is OLS rather than Poisson, because OLS identifies

$$\hat{\eta} = \frac{\Delta \mathrm{Cov}_\Gamma \left(\frac{1}{C_{jt}}, \log(X_{j0}) \right)}{\mathrm{Var}_\Gamma[\log(X_{j0})]}.$$

That is, the OLS shift-share coefficient $\hat{\eta}$ measures how the covariance changes after the fertilizer price shock, averaged over all post-periods. To the extent that baseline fertilizer use X_{j0} correlates strongly with technology α_j, which it does in our model, this regression delivers an approximation of the covariance derivative that allows us to capture the relevant consumption effects for optimal policy.

While returning to the full dynamic model here eliminates some of the theoretical clarity on the relevant channels, the main results will stay the same: there is a non-trivial interplay between the set of instruments available to the planner, the technological structure of the economy, and the level of consumption in relation to it. Moreover, there are several lessons we take into the estimation of the full dynamic model. First, cross-sectional variation in fertilizer use can come from either financial frictions or technology but they enter differently into the optimal policy problem. Financial frictions affect the Domar weights by which the relevant moments of the technological heterogeneity distribution are aggregated ($\mathbb{E}_\Gamma[\alpha]$, $\mathrm{Var}_\Gamma[\alpha]$, etc.). Attributing all variation to financial frictions would eliminate the quantitative bite of the production channel here. The second is the importance of capturing how marginal utilities move with technology and thus captureing outside farming options and wage movement for non-farm income. Our goal next is to link these features of the model back to the reduced-form results in Section 3 and use them to quantify policy.

6 Quantitative Exercise and Model Estimation

6.1 Quantitative Exercise

In our main quantitative exercise, we operate under the assumption that $T_{jt} = 0$ for all village-time pairs.[29] We start the model with a baseline market fertilizer price p_{x0}^{market} and subsidy level τ_{x0} that all agents assume will remain fixed forever. This generates a baseline stationary equilibrium. We then shock the economy with a deterministic sequence $\{p_{xt}^{\mathrm{market}}, \tau_{xt}\}_{t=1}^{t=\infty}$ that eventually converges back to its baseline values $(p_{x0}^{\mathrm{market}}, \tau_{x0})$ and trace

[29] We also run the same Poisson exposure regression as those in Section 3 on the level of government support (measured in RWF) and find no differential effect across villages.

out the transition path of the economy.[30]

We calibrate the model using the empirical counterparts $\{\widehat{p}_{xt}^{\text{market}}, \widehat{\tau}_{xt}\}_{t=0}^{t=\infty}$ that were realized in Rwanda and restrict ourselves to model outcomes from $t \in \{0, 1, 2, 3\}$ to be consistent with our empirics (the shock occurs in the first half of 2022 ($t = 1$), and our data run through June 2024, which is our $t = 3$). Once calibrated, we will hold fixed the exogenous market price $\{p_{xt}^{\text{market}}\}_{t=0}^{t=\infty}$ and feed in different subsidy paths $\{\tau_{xt}\}_{t=0}^{t=\infty}$ to study counterfactual outcomes. Figure 5 plots what we feed into the model exogenously for calibration.

Figure 5: Model Time Paths for Calibration

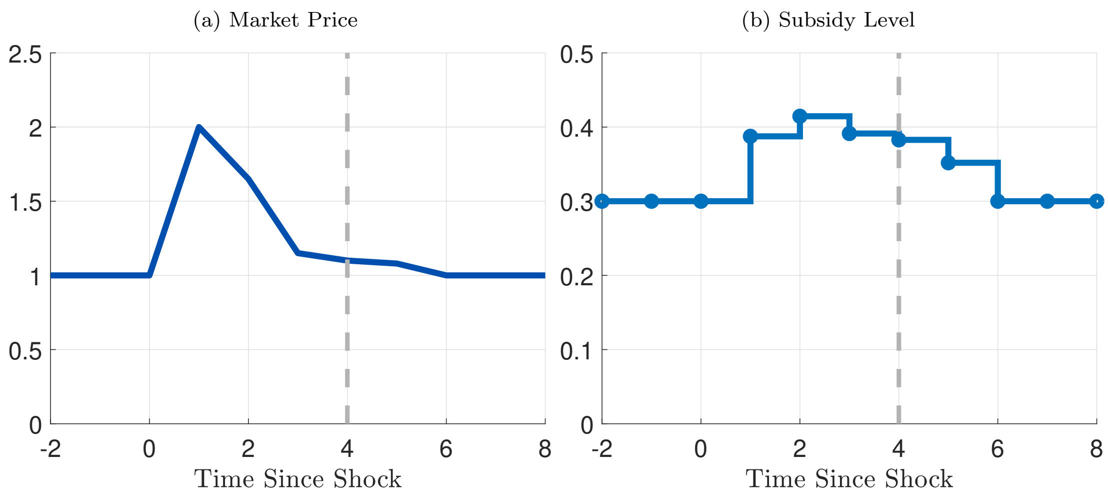

Figure Notes: Panel (a) plots the market price p_x^{market} and Panel (b) the subsidy level τ_x, which is plotted as "constant" within period to match the graphical representation of the earlier empirical results. The gray bar represents when our data collection ends, so the calibration uses only data from the left of that.

6.2 Estimating Model

The model includes several parameters that can roughly be divided into two groups. The first group is either chosen exogenously or matches one-to-one with data moments. The second group is jointly determined by matching equilibrium moments. We discuss these in turn.

6.2.1 Parameters Set Exogenously

We normalize non-agricultural productivity $A = 1$. We also set four parameters outside the model, the discount rate $\beta = 0.95$, the returns to scale $\gamma = 0.8$, and utility parameters $\chi = 2$ and $\zeta = 0.01$.

[30]The international fertilizer price and Rwandan subsidy level were extremely stable in the decade leading up to 2021 (see Section 2). While this is of course just one realization of a random variable, the expert forecasts in Figure 1 are consistent with beliefs that the price stability would continue. Beliefs consistent with this result would closely approximate our assumption. This comes at the quantitative cost of missing any increased uncertainty on the evolution of fertilizer prices after the shock.

We also assume that α and ϕ are distributed according to the joint distribution $G(\alpha, \phi)$. We assume that the marginal distribution of α is drawn from a truncated lognormal distribution on (0,1) with parameters $(\mu_\alpha, \sigma_\alpha)$. We assume that the marginal distribution of $1+\phi$ is distributed exponential with parameter λ_ϕ. Finally, we assume they are joined together with a Clayton copula, which gives the cdf $G(u_\alpha, v_\phi) = \max\{u_\alpha^\rho + v_\phi^\rho - 1, 0\}^{\frac{-1}{\rho}}$ where u_α and v_ϕ are the relevant marginal cdfs and ρ controls dependence.[31] This implies 4 parameters to estimate for the joint distribution.

6.3 Using the Empirical Results to Inform Remaining Calibration

We then make use of our empirical results to choose parameters. Given that our empirics are based on shift-share coefficients, we will write X_{j0} as the total fertilizer expenditure at village j at the baseline steady state, so that our exposure measure in village j is $\log(X_{j0})$.

Substitutability Between Village Production, ν The market clearing condition for the final agricultural good producer implies that village j output price is a function of its production share and the agricultural consumption price, $p_{ajt} = (Y_{at}/y_{ajt})^{\frac{1}{\nu}} p_{act}$. Some algebra implies

$$(1-\nu)\Delta_t \log(p_{ajt}) = \Delta_t \log(p_{ajt} y_{ajt}) - (\Delta_t \log(Y_{at}) + \nu \Delta_t \log(p_{act}))$$

Notice that the last term in parenthesis is constant across villages, which means that

$$(1-\nu)\underbrace{\frac{\partial \Delta_t \log(p_{jt})}{\partial \log(X_{j0})}}_{[+0.030]} = \underbrace{\frac{\partial \Delta_t \log(p_{jt} y_{jt})}{\partial \log(X_{j0})}}_{[-0.041]}. \tag{6.1}$$

The parameter ν depends on the relative movements of village j production to its output price. But more practically, these two terms are explicit formulas for Poisson regression coefficients in a shift-share, the same we measured empirically in Section 3. Those values are given in brackets below the equation, and imply $\nu = 2.37$.

Our theoretical results in Section 4 highlight the importance of this parameter. The natural experiment combined with micro data allows us to infer it independently of the remaining model structure. This immediately tells us that the productive reallocation channel in the model will push down the subsidy after a shock. What remains then is the importance of consumption redistribution.

[31] We choose this particular way to link the two marginals because the Clayton copula has nonzero lower-tail dependence, meaning that we can push our economy toward the lower limit in the (α, ϕ) space without sacrificing variation, while maintaining the simplicity benefits offered by the wider class of Archimedean copulas. We have experimented with different marginal and joint distributions with limited change in quantitative magnitudes.

Production Elasticity σ The first way households can adjust to a shock is by substitute fertilizer for labor within a given technology. How important this is depends on the elasticity of substitution between fertilizer and labor, σ. Conditional on using the modern technology, the cost ratio for household i in village j is

$$\frac{p_{xt}x_{ijt}}{w_{ajt}n_{ijt}} = \frac{\alpha_j}{1-\alpha_j}\left(\frac{w_{ajt}}{p_{xt}}\right)^{\sigma-1}.$$

Denoting S_{jt} as the share of households that use fertilizer in village j at time t, a similar procedure to above yields the Poisson shift-share form again

$$\underset{[-0.057]}{\frac{\partial\Delta_t \log\left(\mathbb{E}_i\left[\frac{p_{xt}x_{ijt}}{w_{ajt}n_{ijt}}\right]\right)}{\partial \log(X_{j0})}} = \underset{[-0.059]}{\frac{\partial\Delta_t \log(S_{jt})}{\partial \log(X_{j0})}} + (\sigma-1)\underset{[+0.068]}{\frac{\partial\Delta_t \log(w_{ajt})}{\partial \log(X_{j0})}}, \tag{6.2}$$

where $\mathbb{E}_i$ denotes an average over households within a village. The empirical values are in brackets below their respective terms (the left-hand side cost ratio includes payments to own farm work in the wage bill). Together, they imply $\sigma = 1.03$.

Aside from realism, these results highlight why including the extensive margin matters here. Without it, the first term on the right is zero, and production function estimates are biased toward complementarity. Our empirics would counterfactually point us to $\sigma = 0.16$.

Non-Agricultural Labor Ability θ The second margin of adjustment is the ability to work off-farm. This, in part, depends on how wages respond. We make use of the properties of the Pareto distribution assumption for non-agricultural labor efficiency units. This implies that a household's total non-agricultural earnings $e^m_{ijt} = (\theta/(\theta-1))\, w_{ajt}^{1-\theta}$, which implies

$$\underset{[-0.108]}{\frac{\partial\Delta_t \log(\mathbb{E}_i[e^m_{ijt}])}{\partial \log(X_{j0})}} = (1-\theta)\underset{[+0.068]}{\frac{\partial\Delta_t \log(w_{ajt})}{\partial \log(X_{j0})}}. \tag{6.3}$$

Using our Poisson empirical results in the same way as before, we estimate $\theta = 2.59$.

Measuring Distortions Like in any model of misallocation, the risk here is that we misattribute variation to technology differences to distortions or vice versa. Our solution is to measure model-consistent distortions as part of our calibration. To operationalize this idea, define λ_{ijt} as the shadow value of in household i in village j at time t, which implies $1+\lambda_{ijt}$ is the reduced-form distortion faced by the household. For households using fertilizer, the first order conditions imply[32]

$$(1+\lambda_{ijt})^{\sigma} p_{xt} x_{ijt} = \gamma^{\sigma} \alpha_j z_{ijt}^{\frac{\sigma-1}{\gamma}} (p_{ajt} y_{aijt})^{\sigma - \frac{\sigma-1}{\gamma}} p_{ajt}^{\frac{\sigma-1}{\gamma}} p_x^{1-\sigma}. \tag{6.4}$$

This is the same moment whose evolution across villages was formalized in the simpler static model of Section 4 (see Proposition 1). Our goal is to measure it using observables. Using the same approach as before with a slight order of operations change,

$$\frac{\partial \Delta_t \mathbb{E}_{ij}[\log(1+\lambda_{ij})]}{\partial \log(X_{j0})} = \left(-\frac{1}{\sigma}\right) \underset{[-0.053]}{\frac{\partial \Delta_t \mathbb{E}_{ij}[\log(p_x x_{ij})]}{\partial \log(X_{j0})}} + \left(\frac{\sigma-1}{\sigma\gamma}\right) \underset{[+0.030]}{\frac{\partial \Delta_t \mathbb{E}_{j}[\log(p_{aj})]}{\partial \log(X_{j0})}} \tag{6.5}$$

$$+ \left(1 - \frac{\sigma-1}{\sigma\gamma}\right) \underset{[-0.076]}{\frac{\partial \Delta_t \mathbb{E}_{ij}[\log(p_{aj} y_{aj})]}{\partial \log(X_{j0})}} + \left(\frac{\sigma-1}{\sigma\gamma}\right) \frac{\partial \Delta_t \mathbb{E}_{ij}[\log(z_{ij})]}{\partial \log(X_{j0})}$$

Despite the cumbersome notation, (6.5) simply tells us how the model-consistent distortion evolves with baseline fertilizer intensity. There are two unobservable terms here: the distortion on the left-hand side and the average modern ability on the right-hand side. However, because we estimate $\sigma = 1.03$, the distortion moment is well-approximated by only observables. Thus, we can target it directly in our calibration, though the subtle change in order of operations – log then average over household-villages – implies that the model-consistent regression is OLS rather than Poisson, and we report the values in (6.5) in from this regression.[33]

Unlike the previous results, measuring distortions in this way does not pin down any parameters directly, because their magnitude will depend on other features of the economy like the persistence and distributional assumptions on productivity. As such, we will use them as moments in the remaining calibration below, allowing us to jointly target the total cross-sectional variation in fertilizer and model-consistent distortions. Roughly, this means targeting the distortion moments directly with financial frictions, then allowing the marginal distribution of technology to soak up the remaining cross-sectional variation in fertilizer use.

[32] While the CES assumption makes it somewhat less intelligible, note that with $\sigma = 1$ it simplifies to

$$\frac{p_{xt} x_{ijt}}{p_{ajt} y_{aijt}} = \frac{\gamma \alpha_j}{1+\lambda_{ijt}}$$

where $\gamma\alpha_j$ is the exponent on fertilizer in the production function $y = z(x^{\alpha} n^{1-\alpha})^{\gamma}$. (6.4) is just the CES generalization of this familiar Cobb-Douglas result.

[33] In Appendix C we use the Pareto properties of modern productivity to develop a more involved strategy that allows us to use the same intuition in the generic CES case, but it offers little additional quantitative value here given our estimate of σ.

6.3.1 Remaining Calibration

This leaves 10 parameters to choose, which we jointly calibrate to match a set of baseline cross-sectional and time series moments.

We set the subsistence level to match the agricultural consumption share of 63 percent, which implies $\overline{a} = 0.67$. We match the gross interest rate $R = 1.01$ so that aggregate consumption to GDP is 90 percent.

Joint Technology-Financial Friction Distribution We then use the parameters in the joint distribution of technology α and financial frictions ϕ to match four moments. The first two relate to fertilizer expenditures. We match (1) the Domar-weighted expected fertilizer share of harvest and (2) the log difference between the 95th an 5th percentiles of fertilizer expenditure per acre (our source of variation in the shift-share results). The last two moments relate to frictions. We match (3) the evolution of distortions as defined in (6.5). Our final moment is the parameter $\hat{\gamma}$ from the OLS regression

$$\log(c_{ijt}^{-2}) = \gamma(\log(X_{j0}) \times Post_t) + \theta_j + \beta_t + \varepsilon_{ijt}$$

with weights defined by the baseline Domar weights and c_{ijt} is total consumption expenditures for household i in village j at time t. The rationale for this moment is discussed in Section 5. It measures the change in Domar-weighted covariance normalized by baseline variance, $\hat{\gamma}$ is

$$\hat{\gamma} = \frac{\mathbb{E}\Big[\Delta_t \mathrm{Cov}_\Gamma(C_{jt}^{-2}, \log(X_{j0}))\Big]}{\mathrm{Var}_\Gamma([\log(X_{j0})]},$$

which governs how the evolution of consumption is affected by policy. The regression averages over post-periods. This is not exactly marginal utility in our model (which would be the exact counterpart to the theory), because it does not include payments to subsistence $p_{ct}\overline{a}$. While easy to define in the model, mapping that to the data is not quite as natural, hence we match this similar but not identical moment instead.

Taken as a whole, this procedure is essentially the following: match how the distortions vary directly and how they affect consumption. Then use the remaining technological variation to soak up remaining variation in fertilizer use across villages.

Fertilizer and Labor-Intensive Technologies Finally, the last four moments relate to the idiosyncratic productivity process and technology choice. We have the minimum value $\underline{z}_M$ and shape parameter θ_M of the Pareto modern sector productivity, the traditional technology productivity A_T, and the persistence ϕ of modern productivity draws. These match four moments. The first is that 58 percent of households use fertilizer at baseline. The second is the baseline relative value of harvest yields between those who do and do not use fertilizer.[34] For the latter, we strip out village-level differences when we estimate, so that our moment is the coefficient $\hat{\beta}$ from the regression

$$\log(y_{ij0}) = \gamma_j + \beta \mathbb{1}[\text{use fert}] + \varepsilon_{ij0}$$

This yields $\hat{\beta} = 0.80$. We also match two time series moments. The first is the persistence of fertilizer use, which we measure with the lagged regression

$$\mathbb{1}[\text{use fert}]_{ijt} = \gamma_j + \theta_t + \beta \mathbb{1}[\text{use fert}]_{ij,t-1} + \varepsilon_{ijt}$$

We find $\hat{\beta} = 0.41$ which requires a persistence parameter of $\psi = 0.47$.[35] Our final moment is how the share of households that use fertilizer evolves with baseline fertilizer expenditure. That is, we match the Pareto regression from Section 3 on the extensive evolution after the shock, where the shift-share implied a coefficient of -0.059 on baseline fertilizer expenditures. That is, households that used more fertilizer at baseline exhibited larger declines on the extensive margin.

Table 4 summarizes our full set of parameters.

[34] In the Cobb-Douglas limit of modern technology, we have that $\mathbb{E}_j(S_{j0}) \propto \frac{z_M}{A_T}$. Thus, we first get the ratio of the two, then use the yield difference to distinguish them. This second moment is decreasing in the ratio z_M/A_T

[35] Interestingly, this low persistence in agriculture productivity has been found in other contexts using either lagged harvest persistence (Donovan, 2021) or production function estimation techniques (Manysheva, 2022), suggesting differences in both steady state costs of misallocation and transition paths relative to sectors with more persistent shocks.

Table 4: Calibration Summary

Parameter	Description	Parameter Value	Moment	Target Value	Model Value
Set Exogenously:					
β	Discount factor	0.95	1 year period	0.95	0.95
χ	CRRA parameter when $\overline{a}=0$	2	Standard value	2	2
ζ	Agricultural utility weight	0.01	Long-run 1% expenditure share on agriculture	0.01	0.01
γ	Returns to scale	0.8	Standard value	0.8	0.8
Match Directly to Empirics:					
ν	Agricultural final good aggregator	2.37	Eq. (6.1) in text	2.37	2.37
σ	Modern tech. CES parameter	1.03	Eq. (6.2) in text	1.03	1.03
θ	Shape, non-agr skills	2.59	Eq. (6.3) in text	2.59	2.59
Jointly Chosen to Match Baseline Model Moments:					
$\overline{a}$	Agricultural subsistence	0.67	Avg. agricultural expenditure share	0.63	0.63
μ_α	Mean CES fertilizer parameter	0.03	Avg. village fertilizer expenditure share of harvest value	0.07	0.07
σ_α	Standard deviation of CES fertilizer parameter	1.01	95-5 ratio of village log fertilizer expenditures per acre	3.55	3.55
R	Gross interest rate on savings	1.01	Consumption as share of GDP	0.90	0.91
$\underline{z}_M$	Minimum value, modern productivity	0.83	Baseline share of HHs using fertilizer	0.58	0.58
A_T	Relative productivity of traditional agriculture	1.02	Baseline log(yield) gain among fertilizer users	0.80	0.83
Jointly Chosen to Match Time Series Model Moments:					
λ_ϕ	Financial friction parameter	11.3	Evolution of covariance of marginal utility and fertilizer use	-0.05	-0.06
ρ	Copula parameter	0.01	Eq. (6.5): $\partial\Delta$ distortion$/\partial\log(X_{j0})$	$\approx$ -0.03	-0.03
θ_M	Shape parameter, modern productivity	4.15	$\partial\Delta_t\mathbb{E}_j\left[\log(S_{jt})\right]/\partial\log(X_{j0})$	-0.059	-0.06
ψ_z	Persistence, modern productivity	0.47	Lagged persistence of fertilizer use	0.41	0.41

Table notes: The $\approx$ symbols in the target values for λ_ϕ and ρ are the Cobb-Douglas approximate values, with the exact method given in the text.

7 Quantitative Results

We first discuss optimal policy and highlight key features of the model that generate the optimal time path of the subsidy. Since we find that the optimal subsidy is below the baseline realized subsidy level in 2020, Section 7.1 quantifies the welfare implications of the optimal transition path.

The time path of the optimal subsidy (again, in the economy with no ex post transfers) is in Figure 6 and includes what we observe in the data. The optimal path starts at 10 percent and remains so after the shock. This differs in two ways from what we see in the data. First, the empirical subsidy rises by 50 percent. Second, the baseline level differs substantially.

Figure 6: Optimal Fertilizer Subsidy Time Path

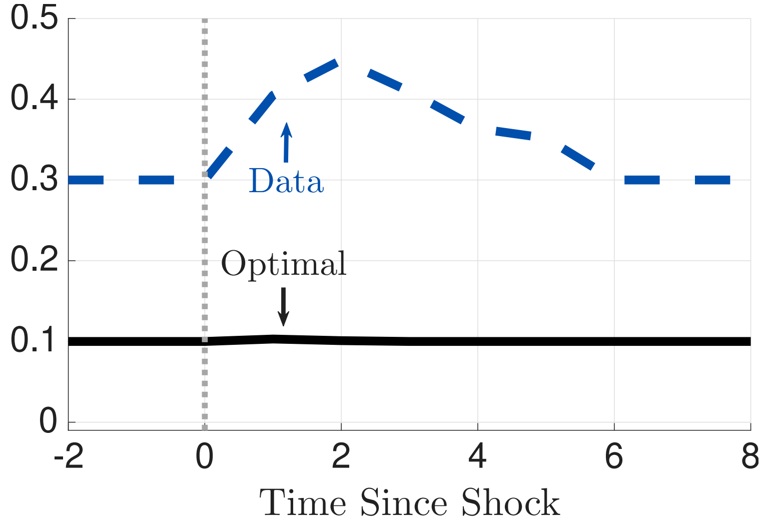

Figure Notes: Panel (a) plots the optimal time path for the fertilizer subsidy, along with the empirically-observed path. Panel (b) overlays two counterfactual time paths for comparison.

Offsetting Effects in the Optimal Subsidy The reason this subsidy is essentially flat is because there are two offsetting effects: the negative pull from the planner's desire to shift production away from high fertilizer villages – driven by our estimation of $\nu > 1$ – and the positive push required to shield relatively poor households in fertilizer-intensive villages. These two turn out to roughly offset.

To see this more clearly, we replace our utility function with a more standard CRRA utility to match the same agricultural consumption share. That is, we replace our baseline model with agricultural utility weight $\zeta = 0.01$ and subsistence requirement $\overline{a} = 0.67$ with $(\zeta, \overline{a}) = (0.63, 0)$. We then recompute the optimal subsidy path. The results are in Figure 7.

Instead of a flat subsidy of 10 percent, the new economy implies an optimal subsidy that falls from 6 to -4 percent on impact. By relying on subsistence requirements to generate

Figure 7: Optimal Path with Alternative CRRA Utility Function

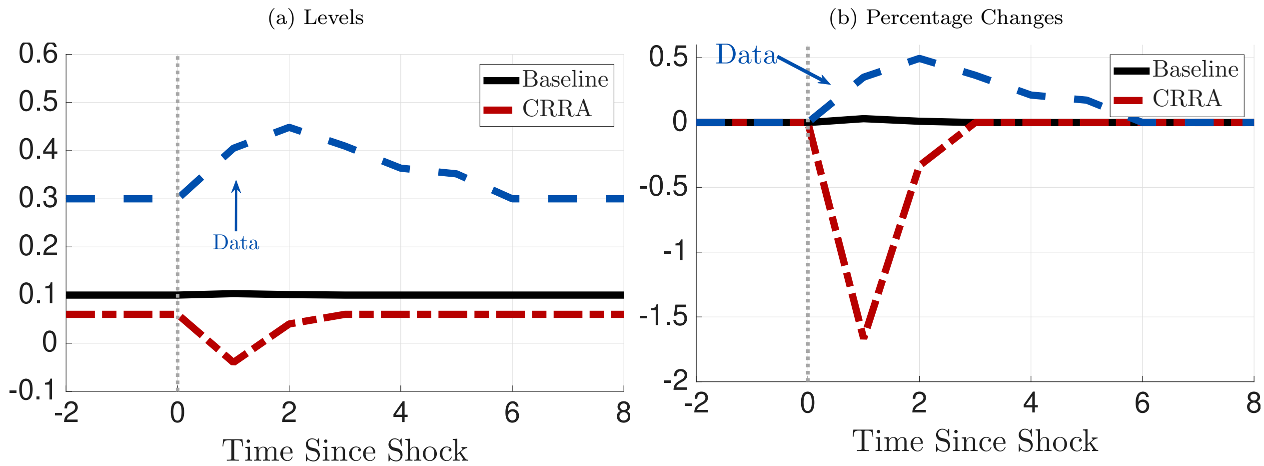

Figure Notes: Plots the baseline model optimal subsidy path and data path. The CRRA plot replaces our baseline utility parameters $(\zeta, \overline{a}) = (0.01, 0.67)$ with $(\zeta, \overline{a}) = (0.63, 0)$, which matches the same agriculture share of consumption expenditures.

agricultural consumption, we give the model the best chance to generate a large change in the optimal subsidy. Even so, it rises by only 3 percent. This highlights the important negative quantitative bite of production reallocation here.

Rationalizing the Low Baseline Level and Small Change The previous results highlight why we do not see a large negative change in the subsidy. We next investigate what features of the model generate the small quantitative magnitude we observe.

Figure 8 plots the optimal subsidy path, along with two counterfactual paths.

Figure 8: Counterfactual Paths

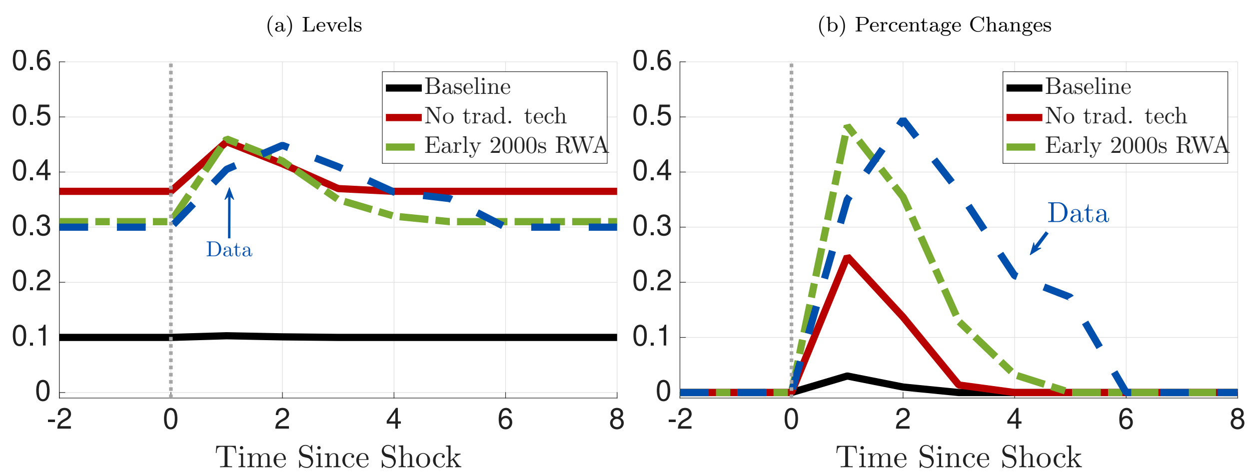

Figure Notes: Plots the optimal subsidy path for two counterfactual economies, along with the baseline economy and data. The first counterfactual economy sets the productivity of non-fertilizer production to $A_T = 0$. The second adjusts $(\overline{a}, R) = (0.69, 0.9)$ from baseline $(0.67, 1.01)$ to mimic a more agriculture-dependent economy with lower savings returns.

The first removes the traditional technology, setting $A_T = 0$. This eliminates the ability of households to substitute into the traditional, fertilizer-free technology and causes both the baseline level and post-shock change to rise. The rationale for both is the same: the outside option helps buffer consumption in the event of a fertilizer price shock. Removing it makes households poorer, and thus the planner more sensitive to their consumption. From the perspective of the data, this counterfactual economy of course cannot match the large extensive margin adjustment we observe after the shock. Thus, our model builds in a relatively elastic extensive margin. This implies an aggregate elasticity much larger than would be estimated from our Cobb-Douglas fertilizer technology.

Our second counterfactual asks whether observed policy would have been closer quantitatively to our model-implied optimal policy in an earlier period. To do so, we adjust $\overline{a}$ from 0.67 to 0.69 and lowers the gross return to savings from $R = 1.01$ to $R = 0.9$. Together, these imply lower savings returns and a agricultural share of consumption of 70 percent, as found in the 2003 Integrated Household Living Survey, the main household survey carried out in Rwanda. In this case, the model closely matches the observed data in baseline level and percentage change after the shock. It rises from 31 percent at baseline to 46 percent on impact, though our model subsidy naturally adjusts immediately on the shock realization.

Recent Rwandan policy deliberations seem to reinforce this view. High baseline subsidies and large positive changes in response to a shock are close to optimal from a historical perspective, but after more than a decade of 9 percentage average labor productivity growth in the sector, likely no longer are. Indeed, recent strategic planning documents highlight an agreement to gradually lower subsidies (World Bank, 2022, 2025) and the government commissioned a series of reports exploring its feasibility. But these reports also highlight the difficult political economy problem of reducing subsidies (IRDP, 2022; Spielman et al., 2025). The time consistency issues that plague industrial policy globally (see Juhász and Lane, 2024, for a review) seem equally at play here. Understanding the constraints inherent in this process, especially when governments use a single tool to achieve longer-run development and short-run stabilization goals, is an important avenue for future work.

7.1 Optimal Transition Path

Motivated by this last set of results, we use the model to measure the optimal transition path starting from $\tau = 0.30$. We assume the fertilizer price shock remains. Perhaps unsurprisingly given the model's optimal subsidy path, the model transition bears little resemblance to the proposed slow gradual fadeout. Instead, it prefers an immediate collapse of the subsidy from 30 to 10 percent.

We compare the optimal policy to other paths in Table 5 The first is the observed subsidy path as implemented by the Rwandan government. The second is a "do nothing" policy, holding the subsidy fixed at 0.30. These generate consumption equivalent welfare gains of 0.9 and 0.6 percent, respectively. For context on the size of these effects, the last row computes a counterfactual economy in which there is no fertilizer price shock but the subsidy remains fixed at 0.30. The transition to optimal policy including the shock generates welfare gains of 0.3 percent. Thus, on average, the economy would be willing to accept the cost of the Russia-induced fertilizer price shock if it came with an optimal subsidy transition.

Table 5: Welfare Gains from Optimal Policy

Scenario	Consumption-Equivalent Welfare (%)
Optimal Transition	–
Subsidy remains at 0.30	0.60
Observed Subsidy Path	0.92
No fertilizer price shock + subsidy remains at 0.30	0.32

When we decompose that 0.3 percent welfare gains, we find that the gains by village vary from 0.1 to 0.5 depending on the combination of technology and financial frictions. However, they are all positive. So even on a village-by-village basis, optimal policy is preferred.

8 Conclusion

How should a planner use a limited set of policy tools in a setting with financial frictions and heterogeneous technology? These are core features of developing countries. We use a quantitative general equilibrium model, primary data, and a large natural experiment that doubles real fertilizer prices in Rwanda to help answer this question. Our results highlight the importance of technological heterogeneity on changing not just the quantitative magnitude but qualitative sign of subsidies after a shock, and the quantitative magnitudes depend on several parameters that govern aggregate elasticities like the availability of alternative "traditional" technologies and the shapes of production function. Though we use Rwandan agriculture as our focus here, both the policy attention and forces at play in our model are general considerations for policy.

Left unexplored here is how these short-term shocks affect long-run implications. We know, for example, that long run elasticities of substitution for intermediates can be substantially different from short-run elasticities (Peter and Ruane, 2022). In fact, in late 2023 the Rwandan government created a joint venture with a Moroccan fertilizer company to create a fertilizer processing plant in Rwanda, though this does not solve problems related to the procurement of natural gas. Alternatives may also include new technologies that deliver nitrogen via different means, such as biofertilizers. To the extent that short-run shocks induce longer-run technological advances, one lesson from this paper is that policy will need to evolve along with it.

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Table of Contents for Online Appendix

A Additional Empirical Results

A.1 Natural Gas Prices and Global Fertilizer Prices

Figure 9 plots the relationship between natural gas prices and fertilizer prices since 2014 from the World Bank Commodity Price Outlook (World Bank, 2024), normalized by January 2014. The correlation between the gas prices and the two fertilizer prices are 0.67 and 0.69 for DAP and urea. Figure 9 also shows that two key fertilizer prices more than double in just two years between 2020 and 2022.

Figure 9: Fertilizer Prices and Natural Gas Prices

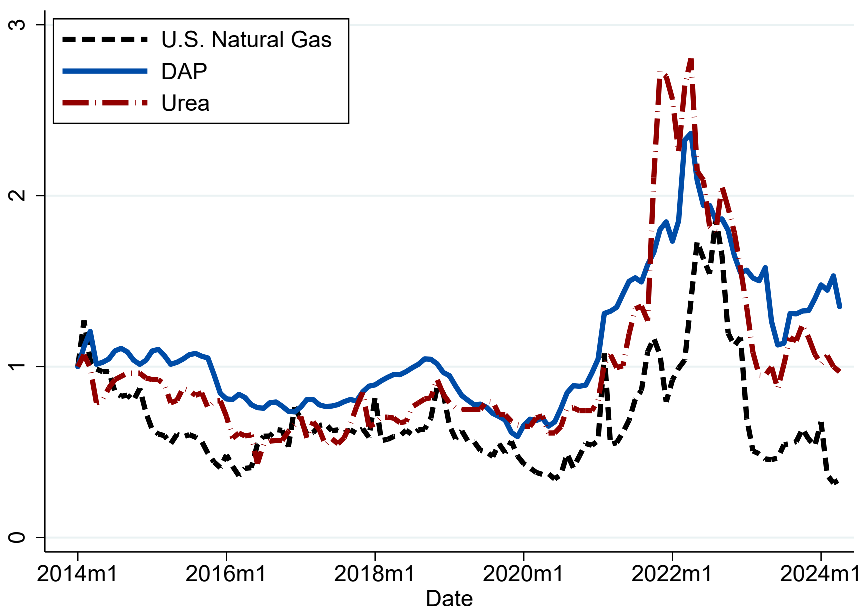


Figure Notes: Nominal monthly prices of DAP, urea, and U.S. natural gas from World Bank (2024). 2014m1 normalized to one.

A.2 Fertilizer versus Other Potential Shocks

Since Russia's invasion of Ukraine affected many potential margins, a natural question is the extent to which the shock we focus on is relevant or co-moves with other substantial shocks. We discuss this issue here.

A.2.1 Additional Data Used Here

In addition to our household level data, in 2021 we began collecting data from traders in the markets associated with these villages. In the 2020 baseline we asked households to tell us about the markets that they bought and sold goods. Any market mentioned by more than 2 households is included in the study, though in practice there are rarely more than one or two markets per village. These data are collected monthly by in-person enumeration. This high-frequency collection requires surveys to be relatively short so that enumerators can travel to many markets over the short time horizon. This survey asks questions about goods stocked, inventory, and prices. We make use of that data here to study prices.

A.2.2 Fertilizer Versus Other Intermediate Inputs

The main argument put forth here is that the concentration of fertilizer in a small subset of countries makes it particularly vulnerable to exogenous variation in things like natural gas markets. Other intermediates – pesticide, fungicide – do not share this feature as they can be more easily sourced locally.

Figure 10 plots the raw log per-unit prices for two main intermediate inputs over time derived from the vendor survey. The first is DAP, a key fertilizer in Rwanda as discussed in the text. The second is the main fungicide used, Roket. The 95 confidence intervals are constructed via standard errors clustered at the market level. Each contains a dashed line at January 2022, where prices are normalized to zero. There is a strong increase in DAP prices and none in fungicide prices.

To study these patterns more systematically we exploit the panel dimension of the data and run a series of regressions

$$\log(p^j_{imt}) = \alpha + \beta Post_t + \theta_i + \eta_m + \nu \log(t) + \varepsilon_{it} \tag{A.1}$$

where, for each product j, we regress the log price on a dummy $Post_t = 1$ if post-January 2022. We control for vendor i's fixed effect θ_i, month effects η_m for seasonality, and $\log(t)$ is a linear time trend. The results are in Table 6 and confirm the patterns from the raw data in Figure 10.

Figure 10: Raw Prices for Inputs (2021-2024)

(a) Fertilizer (DAP) (b) Fungicide (Roket)

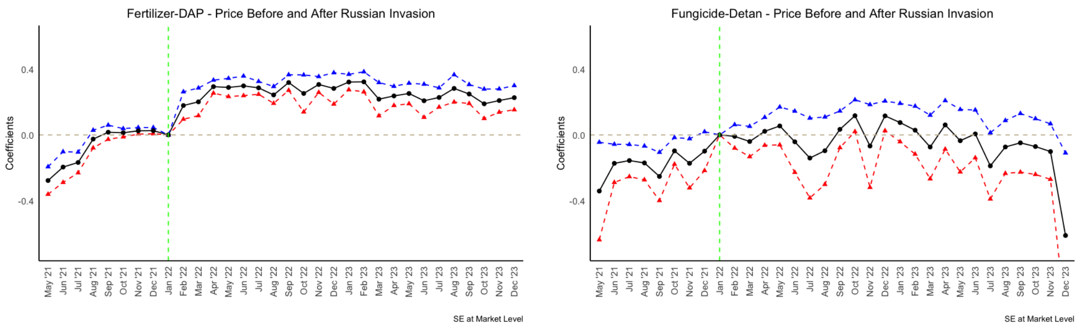


Figure Notes: This figure plots log prices for the listed items over time as averages across all vendors. Dashed lines indicate 95 percent confidence interval with standard errors clustered by market.

Table 6: Price Changes Post-January 2022 for Intermediate Inputs

Inputs	Fertilizer			Other Intermediates		
	DAP	NPK	Urea	Roket	Detan	Ridomil
Post-January 2022	0.35***	0.39***	0.33***	0.41	0.14*	-0.18
	(0.03)	(0.05)	(0.08)	(0.26)	(0.07)	(0.24)
Observations	1,658	1,227	1,605	3,172	2,623	1,192
R-squared	0.44	0.43	0.14	0.61	0.27	0.41

Table notes: Standard errors clustered by market are in parentheses. Ridomil is a fungicide and Rocket and Detan are insecticides used in Rwanda. All regressions include trader and month fixed effects and a linear time trend. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

A.2.3 Fertilizer Versus Exogenous Food Supply Shocks via Imports

Another possibility is that the direct import collapse drives agricultural prices higher. Such an argument would be difficult to reconcile with the shift-share results that price changes vary systematically across space with baseline fertilizer intensity. Regardless, we offer additional direct evidence in this section that suggests the changes to agricultural output prices are linked to changes to fertilizer and domestic agricultural productivity changes.

First, we measure direct exposure to Ukrainian imports in Rwanda. All trade data in this section is take from CEPII's BACI database (Gaulier and Zignago, 2010), January 2025 release. On an aggregate level, Ukraine accounts for almost no imports into Rwanda. Ukraine accounted for 0.1 percent of the total value of imports into Rwanda and 0.6 percent of its agricultural imports (HS17 2 digit codes 01-15). Table 7 shows the main products that Rwanda imports from Ukraine, including the share of Ukrainian imports each accounts for.

Table 7: Share of Import Value from Ukraine in 2020

Product Code	Description	Ukranian Import Share
151219	Vegetable oils: sunflower seed or safflower oil	0.303
721391	Iron or non-alloy steel	0.275
110100	Wheat or meslin flour	0.155
680422	Millstones, grindstones, grinding wheels and the like	0.066
220890	Spirits, liqueurs and other spirituous beverages:	0.065
210210	Yeasts: active	0.052
841210	Engines: reaction engines, other than turbo-jets	0.028
880310	Aircraft and spacecraft: propellers and rotors and parts thereof	0.018
902610	Instruments and apparatus: for measuring or checking the flow or level of liquids	0.011

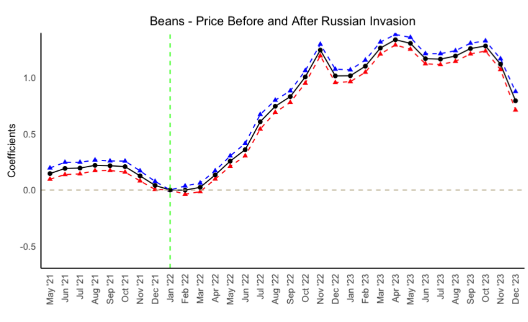
Table notes: This table lists all 6 digit products that make up more than 1 percent of the value of Ukrainian imports into Rwanda in 2020. Product codes are HS17 and a brief description is given in the second column.

The only raw agricultural product imported is wheat, which Ukraine exports globally. It makes up 15.5 percent of Rwandan imports from Ukraine and 40 percent of total Rwandan wheat imports. Wheat is not produced in any serious quantity in Rwanda. In the 2020 Agricultural Household Survey (NISR, 2020), only 3.5 percent of households grow any wheat. No one in our dataset produces wheat. Thus, exposure to wheat shocks is limited here. This contrasts with other countries like Ethiopia (Adamopoulos and Leibovici, 2024).

Figure 11 offers additional evidence from a different angle. It plots the price patterns for two main staple crops that are consumed domestically and not imported. Despite this, both see their prices rise with a few-month lag, suggesting an important equilibrium spillover onto food prices for a domestic productivity shock.

Figure 11: Raw Prices for Local Crops (2021-2024)

(a) Beans



(b) Sweet Potatoes

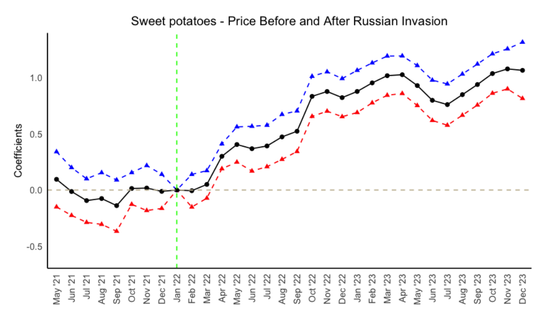


Figure Notes: This figure plots log prices for the listed items over time as averages across all vendors. Dashed lines indicate 95 percent confidence interval with standard errors clustered by market.

A.3 Comparing Our Data to Nationally-Representative Data

About every 3 years, Rwanda conducts its Agricultural Household Survey (AHS), which is representative household data that collects information on agricultural practices. We compare our baseline data in 2020 to the 2020 AHS (NISR, 2020). Figure 12 plots the distribution of hectares cropped across households in the AHS and our dataset; the distributions are quite similar. We also compare our sample on the likelihood of using inorganic fertilizer in panel (b) (the AHS reports only an indicator for inorganic fertilizer use, not quantity or expenditure). Our sample somewhat over-represents fertilizer use relative to the national average, though it matches the log-linear shape with land size quite well.

Figure 12: Comparison to Nationally Representative Data in 2020

(a) Farm Size Distribution (b) Likelihood of Using Inorganic Fertilizer

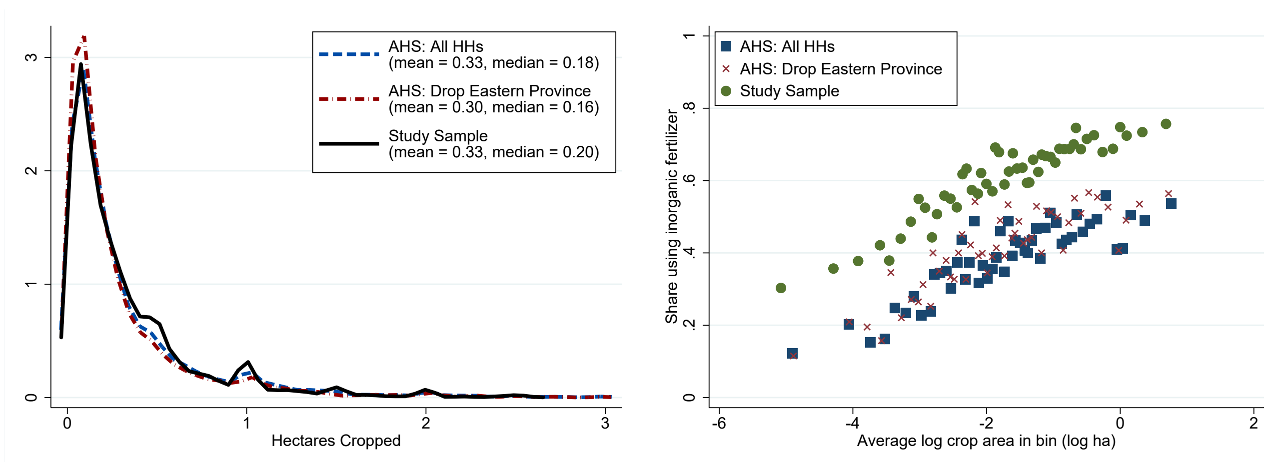


Figure Notes: This figure compares the moments in our baseline study sample in 2020 to nationally representative data from the 2020 Rwandan Agricultural Household Survey (AHS). The top 1 percent of observations are trimmed for farm size. For panel (b), we bin farm size into 30 equally spaced bins and report the averages.

A.4 Market-Level Interactions

Villages do not each sell to unique markets. Figure 13 shows 9 villages (the yellow pins) that interact in two different markets (the red shopping carts). We would only expect the market price to respond to a village's own fertilizer intensity if it made up an important share of the market-level fertilizer intensity. As a robustness check, we therefore measure the residual fertilizer intensity of the market by computing the average fertilizer intensity of market-connected villages, given by (A.2), and test how prices adjust.

Figure 13: Example of Village Links to Markets

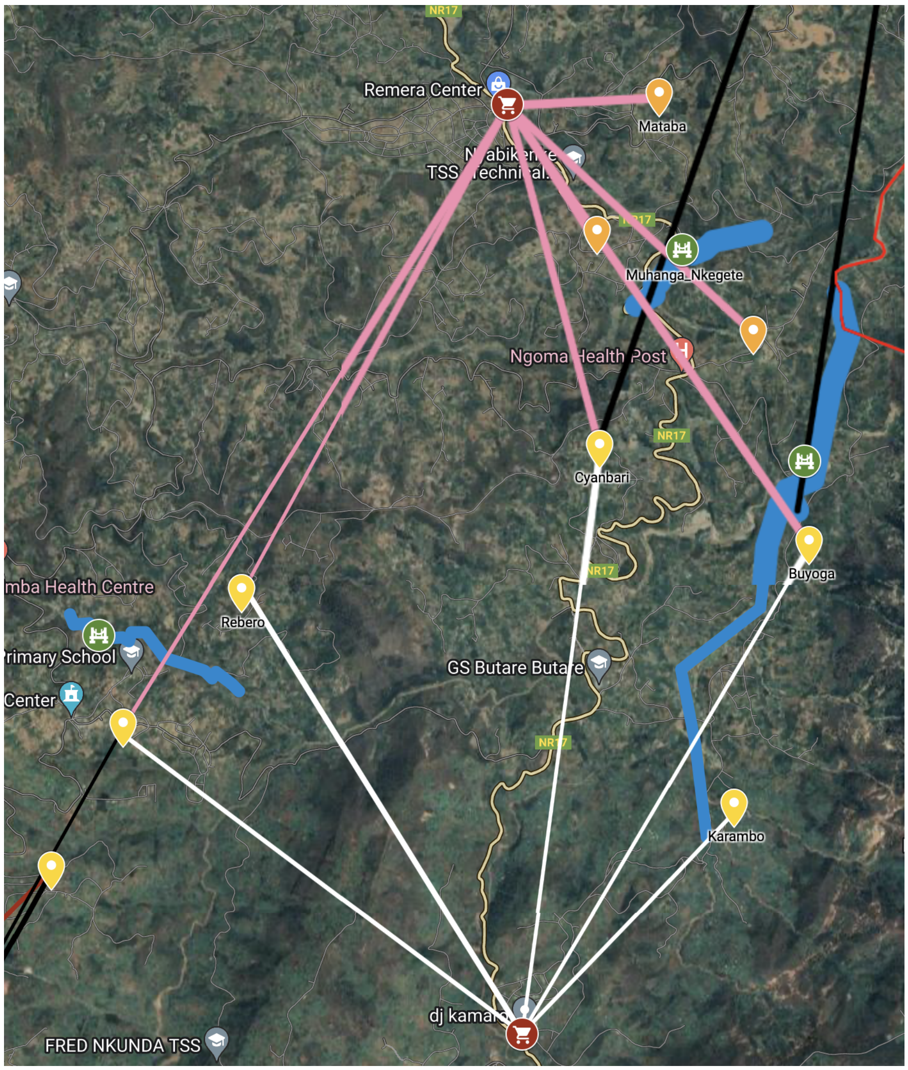


Table 8: Crop Prices

VARIABLES	All (1)	All (2)	Low Market (3)	High Market (4)
Post	0.023***	-0.074***	-0.078***	-0.061***
	(0.009)	(0.014)	(0.020)	(0.023)
Post x Village Fertilizer Intensity		0.032***	0.032***	0.012
		(0.006)	(0.009)	(0.010)
Observations	59,150	59,145	30,524	28,621

Table notes: Standard errors clustered by village are in parentheses. All outcome variables are in logs and regressions are run as OLS. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

$$\tilde{f}_{Mv0} = \log\left(\frac{\overbrace{\sum_{\hat{v}\in M_v}\sum_{h\in\hat{v}} p_{x0} f_{h\hat{v}0}}^{\text{fert. for hhs that trade in same market}} - \overbrace{\sum_{h\in v} p_{x0} f_{hv0}}^{\text{own fert.}}}{\underbrace{\sum_{\hat{v}\in M_v}\sum_{h\in\hat{v}} \ell_{h\hat{v}0}}_{\text{land for hhs that trade in same market}} - \underbrace{\sum_{h\in v} \ell_{hv0}}_{\text{own land}}} \right) / |M_v| \tag{A.2}$$

(A.2) takes total baseline fertilizer for all households that trade in the same market as village v net of that fertilizer from v, and similarly for land, to compute a net fertilizer intensity for the market. If $\tilde{f}_{Mv0}$ is high, then village v makes up only a small portion of the market's fertilizer intensity. We would not expect output prices to respond to village v's fertilizer intensity in this case. On the other hand, if $\tilde{f}_{Mv0}$ is low, village v plays an important role in market M.

Columns (3) and (4) replicate the same regression as Column (2) of Table 2, but breaks the sample into those with high market-connected fertilizer intensity and those with low market-connected fertilizer intensity. The output price in v responds to v's fertilizer intensity only when v is a sufficiently large share of the market.

B Competitive Equilibrium of Simplified Model

Given a subsidy level τ_x and market price of fertilizer p_x^{market}, a competitive equilibrium of this model is a set of decision rules for households $(c_{aj}, c_{mj}, x_j, n_{aj})$, the manufacturing firm N_m, and the final agricultural good producer y_{aj}, prices $\{p_{aj}\}, p_{ac}, w_a$ and tax rate τ_m such that (i) the household's decision rules are consistent with its optimization problem given prices and taxes, (ii) N_m solves the manufacturing firm's decision problem, (iii), y_{aj} solves the agricultural final goods firm decision problem, and (iv) the government balances its budget and markets clear:

1. Government budget balance: $\tau_x p_x^{market} \int_j x_j dj = \tau_m \int_j c_{mj} dj$.
2. Net zero transfers $\int_j T_j dj = 0$
3. Market clearing:
   (a) Agricultural intermediate goods market: for each j, $y_{aj} = x_j^{\alpha_j} n_j^{\eta_j}$
   (b) Agricultural final goods market: $\left(\int_j y_j^{\frac{\nu-1}{\nu}} dj\right)^{\frac{\nu}{\nu-1}} = \int_j c_{aj} dj$
   (c) The manufacturing labor market: $N_m = 1 - \int_j n_j dj$, which amounts to the condition that $w_a = w_m = A$.

C Additional Quantitative Results

C.1 General CES Procedure for Estimating Model-Consistent Distortions

These equations have two important features. First, the right-hand side terms are *almost* all directly observable, except for the change in productivity of modern farmers, a complication induced by technology choice in non-Cobb-Douglas settings (though our estimate of $\sigma = 1.03$ implies almost no weight on this term). Second, there is a subtle difference in the ordering of operations. Here we take logs first before averaging, so the model-consistent regressions are OLS instead of Poisson. Those coefficients are reported under the relevant terms.

Our goal is to use (6.5) and (6.5) as targets in calibration so that we can directly target distortions. $\sigma = 1.03$ implies that the main source of variation comes from the observable terms related to fertilizer expenditures and harvest values, as expected in a Cobb-Douglas setting. Under exactly Cobb-Douglas, we have $\frac{\partial \Delta_t \mathbb{E}_{ij}[\log(1+\lambda_{ij})]}{\partial \log(X_{j0})} = -0.027$. But we do not need to resort to an approximation. We can overcome the un-observability of the final selection term by appealing to the Pareto properties of modern ability, which we summarize in Proposition 3.[36]

Proposition 3. *The expected log productivity of modern farmers is related to the expected share of households that use the modern technology S_{jt} by the relationship*

$$\Delta_t \mathbb{E}_{ij}[\log(z_{ij})|\textit{use modern tech}] = \left(\frac{-1}{\theta_M}\right) \Delta_t \mathbb{E}_j[\log(S_j)], \tag{C.1}$$

and therefore

$$\frac{\partial \Delta_t \mathbb{E}_{ij}[\log(z_{ij})|\textit{use modern tech}]}{\partial \log(X_{j0})} = \left(\frac{-1}{\theta_M}\right) \underset{[-0.059]}{\frac{\partial \Delta_t \mathbb{E}_j[\log(S_j)]}{\partial \log(X_{j0})}} \tag{C.2}$$

This result, which fully accounts for how borrowing constraints evolve with the shock, offers us a way to target distortions directly for any given modern productivity shape parameter θ_M because we can observe changes to the extensive margin of fertilizer use.

Intuitively then, our full strategy is as follows. First, we pick θ_M. We then simulate our model until we match 10 moments, which includes the distortion moment that makes use of

[36] Practically, assuming Cobb-Douglas directly saves little time in the estimation procedure. Doing so guarantees only that the distortion moments do not vary with our parameter vector guesses. It does not change the number of parameters to calibrate. Regardless, (6.5) and (6.5) vary little with changes to parameter guesses given $\sigma = 1.03$.

(C.2). Finally, we check whether or not the implied model is consistent with the empirical variation in modern technology use across villages, $\partial \Delta_t \mathbb{E}_j \left[\log(S_{jt})\right] / \partial \log(X_{j0}) = -0.059$, implying that our parameters are internally consistent with the variation used to measure distortions.

D Proofs

D.1 Characterization of Frictionless Equilibrium and Proof of Lemma 1

Proof. The proof of Lemma 1 proceeds essentially by guess-and-verify. Specifically, we hypothesize that there exists some value $\mathcal{Y}$ such that the following results hold:

$$\frac{p_{ac}^2}{p_{ac}^1} = \frac{Y_a^1}{Y_a^2} = \mathcal{Y}$$

We then show that there is only one value for $\mathcal{Y}$ that satisfies the market clearing conditions and optimality conditions.

Constructing the Candidate $\mathcal{Y}$ From market clearing for village production

$$p_{aj} = \left(\frac{Y_a}{y_{aj}}\right)^{\frac{1}{\nu}} p_{ac}$$

Therefore,

$$\frac{p_{aj}^2}{p_{aj}^1} = \left(\frac{Y_a^2}{Y_a^1}\right)^{\frac{1}{\nu}} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-1}{\nu}} \left(\frac{p_{ac}^2}{p_{ac}^1}\right) = \mathcal{Y}^{\frac{\nu-1}{\nu}} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-1}{\nu}}. \tag{D.1}$$

Using this price ratio in combination with the village farm first order conditions implies

$$\frac{y_{aj}^2}{y_{aj}^1} = \left(\frac{p_{aj}^2}{p_{aj}^1}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j}{1-\gamma}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu(1-\gamma)}} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-\gamma}{\nu(1-\gamma)}} \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j}{1-\gamma}}$$

which, re-arranging, gives

$$\frac{y_{aj}^2}{y_{aj}^1} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu-(\nu-1)\gamma}} \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j \nu}{\nu-(\nu-1)\gamma}}. \tag{D.2}$$

and plugging this result into (D.1) gives the village price ratio of

$$\begin{aligned}
\frac{p_{aj}^2}{p_{aj}^1} &= \mathcal{Y}^{\frac{\nu-1}{\nu}} \mathcal{Y}^{\frac{-(\nu-1)\gamma}{\nu(\nu-(\nu-1)\gamma)}} \left(\frac{p_x^2}{p_x^1}\right)^{\frac{\alpha_j}{\nu-(\nu-1)\gamma}} \\
&= \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma-1}{\nu-(\nu-1)\gamma}} \left(\frac{p_x^2}{p_x^1}\right)^{\frac{\alpha_j}{\nu-(\nu-1)\gamma}}
\end{aligned} \tag{D.3}$$

Aggregate to get total production

$$Y_2 = \left(\int y_{2j}^{\frac{\nu-1}{\nu}} \, dj \right)^{\frac{\nu}{\nu-1}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu-(\nu-1)\gamma}} \left(\int_j \left(\frac{p_x^2}{p_x^1} \right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} \, dj \right)^{\frac{\nu}{\nu-1}} .$$

Under the maintained hypothesis that $Y_{a2} = Y_{a1}\mathcal{Y}^{-1}$, then

$$\mathcal{Y}^{-1} \left(\int_j y_{a1j}^{\frac{\nu-1}{\nu}} \, dj \right)^{\frac{\nu}{\nu-1}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu-(\nu-1)\gamma}} \left(\int_j \left(\frac{p_x^2}{p_x^1} \right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} \, dj \right)^{\frac{\nu}{\nu-1}}$$

which can be re-arranged to solve for the candidate $\mathcal{Y}$

$$\mathcal{Y} = \left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}} \, dj}{\int_j \left(\frac{p_x^2}{p_x^1} \right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} \, dj} \right)^{\frac{\nu-(\nu-1)\gamma}{\nu-1}} . \tag{D.4}$$

The construction of $\mathcal{Y}$ in (D.4) uses two conditions. First, it uses the relevant farm first order conditions. Thus, it already satisfies the optimality conditions of the village farms. Second, it uses the market clearing condition from the stand-in firm that aggregates village production into the agricultural final good. Thus, it satisfies this market clearing condition by construction. The only remaining non-trivial market clearing condition is that of the demand and supply for the final agricultural good.

Confirming that Agricultural Market Clearing Condition is Satisfied Combining the characterizations of village output quantity and price from equations (D.2) and (D.3) gives farm revenue

$$\frac{p_{a2j} y_{a2j}}{p_{a1j} y_{a1j}} = \mathcal{Y}^{\frac{\nu-1}{\nu-(\nu-1)\gamma}} \left(\frac{p_x^2}{p_x^1} \right)^{\frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}}$$

and therefore farm profit can be written as (note that this equation is used in the main text)

$$\begin{aligned}
\pi_{2j} &= (1-\gamma)p_{a2}y_{2j} \\
&= (1-\gamma)\mathcal{Y}^{\frac{\nu-1}{\nu-(\nu-1)\gamma}}\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} p_{a1}y_{1j} \\
&= \underbrace{\left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}}dj}{\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}dj}\right)\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}}}_{\equiv \text{ adjustment to } \pi_{1j}} \underbrace{(1-\gamma)p_{a1}y_{1j}}_{\equiv \pi_{1j}}
\end{aligned}$$

Using the fact that $p_{aj} = (Y_a/y_{aj})^{\frac{1}{\nu}}p_{ac}$, total baseline income is

$$\begin{aligned}
\int_j \pi_{j1}dj + w &= (1-\gamma)\int_j p_{a1j}y_{a1j}dj + w = (1-\gamma)Y_{a1}^{\frac{1}{\nu}}p_{ac1}\int y_{a1j}^{\frac{\nu-1}{\nu}}dj + w \\
&= (1-\gamma)p_{ac1}\Big(\int_j y_{a1j}^{\frac{\nu-1}{\nu}}\Big)^{\frac{\nu}{\nu-1}} + w = (1-\gamma)p_{a1c}Y_{a1} + w
\end{aligned}$$

which remains constant after the price shock

$$\begin{aligned}
\int \pi_{j2}dj + w &= (1-\gamma)\left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}}dj}{\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}dj}\right)\left(\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} p_{a1j}y_{1j}dj\right) + w \\
&= (1-\gamma)\left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}}dj}{\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}dj}\right)\left(Y_{a1}^{\frac{1}{\nu}}p_{ac1}\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{1j}^{\frac{\nu-1}{\nu}}dj\right) + w \\
&= (1-\gamma)\left(\int_j y_{a1j}^{\frac{\nu-1}{\nu}}dj\right)Y_{a1}^{\frac{1}{\nu}}p_{ac1} + w \\
&= (1-\gamma)p_{a1c}Y_{a1} + w.
\end{aligned}$$

Total agricultural demand is therefore

$$D_{a2} = \frac{\zeta}{p_{ac2}}\left(\int_j \pi_{j2}dj + w\right) = \frac{\zeta}{\mathcal{Y}p_{a1c}}\left(\int_j \pi_{j1}dj + w\right) = \frac{D_{a1}}{\mathcal{Y}}.$$

Since $Y_{a2} = Y_{a1}/\mathcal{Y}$, this implies $Y_{a2} = D_{a2}$ as required. ■

D.2 Proof of Proposition 1

D.2.1 Additional Lemma to Characterize Frictional Equilibrium

Lemma 3. *In the frictional economy with binding financial constraints, the equivalent value to that in Lemma 1 is*

$$\mathcal{Y}^c = \left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}} \, dj}{\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu}} y_{aj1}^{\frac{\nu-1}{\nu}} \, dj} \right)^{\frac{\nu}{\nu-1}}.$$

Proof. This proof proceeds identically to that of Lemma 1. ■

D.2.2 Proof of Proposition 1

Proof. Lemma 3 implies

$$\frac{p_{aj}^2}{p_{aj}^1} = \left(\frac{Y_a^2}{Y_a^1}\right)^{\frac{1}{\nu}} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-1}{\nu}} \left(\frac{p_{ac}^2}{p_{ac}^1}\right) = \mathcal{Y}^{\frac{-1}{\nu}} \mathcal{Y} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-1}{\nu}} = \mathcal{Y}^{\frac{\nu-1}{\nu}} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-1}{\nu}}$$

Plugging in

$$y_{aj}^c = \frac{z_j \phi_j^\gamma}{\gamma^\gamma} \left(\frac{\alpha_j}{p_x}\right)^{\alpha_j} \left(\frac{\eta_j}{w}\right)^{\eta_j}$$

gives

$$\frac{p_{aj}^2}{p_{aj}^1} = \mathcal{Y}^{\frac{\nu-1}{\nu}} \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j}{\nu}}.$$

The village j shadow price of fertilizer at time t is

$$1 + \lambda_{jt} = \frac{\alpha_j p_{aj} z_j x^{\alpha_j - 1} n_a^{\eta_j}}{p_{xt}}.$$

Using the relevant input choices and the prices,

$$\Delta_t \log(1+\lambda_j) = \Delta_t \log(p_{aj}) - \alpha_j \Delta_t \log(p_x) = \frac{\nu-1}{\nu} \log(\mathcal{Y}) + \frac{\alpha_j(1-\nu)}{\nu} \Delta_t \log(p_x).$$

Taking two derivatives gives the result

$$\frac{\partial^2 \Delta_t \log(1+\lambda_j)}{\partial \Delta_t \log(p_x) \partial \alpha_j} = \frac{1-\nu}{\nu}.$$

■

D.3 Proof of Proposition 2

D.3.1 An Additional Lemma

Before proving this result, it is helpful to characterize a few results from the frictional equilibrium. We do those in Lemma 4.

Lemma 4. *Two results are true in any equilibrium of the frictional economy.*

1. *Total economy-wide expenditures* $\overline{C} := \int_j \pi_j dj + A$ *do not depend on the market price* p_x^{market} *or the subsidy* τ_x.

2. *The budget-balancing manufacturing tax* τ_m *does not depend on market price* p_x^{market}.

Proof.

Claim 1 Using the result that $p_{aj} = (Y_a/y_j)^{\frac{1}{\nu}} p_{ac}$, total farm profits are

$$\begin{aligned}\int_j \pi_{j1} dj &= \int_j p_{a1j} y_{a1j} dj - \mathbb{E}[\phi] = Y_{a1}^{\frac{1}{\nu}} p_{ac1} \int y_{a1j}^{\frac{\nu-1}{\nu}} dj - \mathbb{E}[\phi] \\ &= p_{ac1} \Big(\int_j y_{a1j}^{\frac{\nu-1}{\nu}} \Big)^{\frac{\nu}{\nu-1}} - \mathbb{E}[\phi] = p_{a1c} Y_{a1} - \mathbb{E}[\phi]\end{aligned}$$

But for any value $p_{x2} = p_{x2}^{\text{market}}(1-\tau_{x2})$, we have $p_{c2}/p_{c1} = Y_{a1}/Y_{a2}$ or $p_{c2}Y_{a2} = p_{c1}Y_{a1}$ as derived in the proof of Proposition 1. Therefore, $\int_j \pi_{j2} dj = p_{1c} Y_{a1} - \mathbb{E}[\phi] = \int_j \pi_{1j} dj$. Since total expenditures are $\overline{C}_2 = \int_j \pi_{j2} dj + A$, we have $\overline{C}_2 = \overline{C}_1$ as required.

Claim 2 The government's budget constraint is

$$\tau_x p_x^{\text{market}} \int_j x_j dj = \tau_m \int_j c_{mj} dj.$$

Optimality conditions imply

$$\begin{aligned}x_j &= \frac{\alpha_j \phi_j}{\gamma p_x^{\text{market}}(1-\tau_x)} \\ c_{mj} &= \frac{(1-\zeta)(\pi_j + A + T_j)}{1+\tau_m}\end{aligned}$$

where π_j is total expenditures in village j. This implies that the government budget constraint is

$$\left(\frac{\tau_x}{1-\tau_x}\right)\frac{1}{\gamma}\Big(\int_j \alpha_j \phi_j dj\Big) = \frac{\tau_m}{1+\tau_m}(1-\zeta)E$$

where $E = \int_j \pi_j dj + A$ is economy-wide expenditures. By the first result that E does not depend on p_x^{market}, this proves the second claim. ■

D.3.2 Proof of Proposition 2

Proof. Denoting $C_j = \pi_j + A + T_j$ as the expenditures of village j and $\overline{C} = \int_j C_j dj$, we can write the consumption decisions $p_{ac}c_a = \zeta C_j$ and $(1+\tau_m)c_m = (1-\zeta)C_j$ which gives the indirect utility function for village j,

$$\Omega - \zeta \log(p_{ac}) + (\zeta - 1)\log(1+\tau_m) + \log(C_j)$$

with constant Ω. Manipulating the agricultural consumption market clearing condition implies

$$p_{ac} = \frac{\zeta \overline{C}}{Y_a}$$

The utilitarian government problem is therefore

$$S := \max_{\tau_x} \; e^{\tilde{\Omega}} Y_a^{\zeta}(1+\tau_m)^{\zeta-1}E^{1-\zeta}$$

or

$$\log(S) = \max_{\tau_x} \; \tilde{\Omega} + \zeta\log(Y_a) + (\zeta-1)\log(1+\tau_m) + (1-\zeta)\log(\overline{C}).$$

where $\tilde{\Omega} := \Omega - \zeta\log(\zeta)$. By Lemma 4, E does not respond to τ_x. Therefore, the first order condition is

$$\zeta\frac{\partial \log(Y_a)}{\partial \tau_x} + (\zeta-1)\frac{\partial \log(1+\tau_m)}{\partial \tau_x} = 0. \tag{D.5}$$

By the implicit function theorem, we are interested in

$$\frac{\partial \tau_x^*}{\partial p_x^{\text{market}}} = -\frac{\frac{\partial^2 \log(S)}{\partial \tau_x \partial p_x^{\text{market}}}}{\frac{\partial^2 \log(S)}{\partial \tau_x^2}}$$

Since $\log(S)$ is concave,

$$\text{sign}\left(\frac{\partial \tau_x^*}{\partial p_x^{\text{market}}}\right) = \text{sign}\left(\frac{\partial^2 \log(S)}{\partial \tau_x \partial p_x^{\text{market}}}\right). \tag{D.6}$$

Taking the derivative of (D.5) and using the results of Lemma 4 to note that $\partial \tau_m / \partial p_x^{\text{market}} = 0$, the sign restriction in (D.6) is equivalent to

$$\text{sign}\left(\frac{\partial \tau_x^*}{\partial p_x^{\text{market}}}\right) = \text{sign}\left(\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}}\right). \tag{D.7}$$

Since

$$y_{aj} = \frac{\phi_j}{\gamma^\gamma}\left(\frac{\alpha_j}{p_x^{\text{market}}(1-\tau_x)}\right)^{\alpha_j}\left(\frac{\eta_j}{A}\right)^{\eta_j},$$

then

$$\log(Y) = \left(\frac{\nu}{\nu-1}\right)\log\left(\int_j \left[\Delta_j (p_x^{\text{market}})^{-\alpha_j}(1-\tau_x)^{-\alpha_j}\right]^{\frac{\nu-1}{\nu}} dj\right)$$

where Δ_j summarizes the remaining terms from y_{aj} that do not include p_x^{market} or τ_x. Taking the derivative with respect to the market fertilizer price,

$$\frac{\partial \log(Y_a)}{\partial p_x} = \frac{-1}{p_x^{\text{market}}}\int_j \alpha_j \frac{\left[\Delta_j (p_x^{\text{market}})^{-\alpha_j}(1-\tau_x)^{-\alpha_j}\right]^{\frac{\nu-1}{\nu}}}{\int_k \left[\Delta_k (p_x^{\text{market}})^{-\alpha_k}(1-\tau_x)^{-\alpha_k}\right]^{\frac{\nu-1}{\nu}} dk}.$$

Define the probability measure

$$\Gamma_j = \frac{\left[\Delta_j (p_x^{\text{market}})^{-\alpha_j}(1-\tau_x)^{-\alpha_j}\right]^{\frac{\nu-1}{\nu}}}{\int_k \left[\Delta_k (p_x^{\text{market}})^{-\alpha_k}(1-\tau_x)^{-\alpha_k}\right]^{\frac{\nu-1}{\nu}} dk}$$

so that we can re-write

$$\frac{\partial \log(Y_a)}{\partial p_x} = \frac{-\mathbb{E}_\Gamma[\alpha; \tau_x]}{p_x^{\text{market}}}$$

where $\mathbb{E}_\Gamma$ is the expectation of α taken using Γ, which depends (in part) on τ_x. This implies

$$\text{sign}\left(\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}}\right) = -\ \text{sign}\left(\frac{\partial \mathbb{E}_\Gamma[\alpha; \tau_x]}{\partial \tau_x}\right). \tag{D.8}$$

Note that

$$\frac{\partial \log(\Gamma_j/\Gamma_k)}{\partial \tau_x} = \frac{\nu-1}{\nu}(\alpha_j - \alpha_k)\left(\frac{1}{1-\tau_x}\right).$$

If $\nu > 1$, this is positive for $\alpha_j > \alpha_k$, implying that weight is shifting toward higher α, and therefore $\partial \mathbb{E}_\Gamma[\alpha; \tau_x]/\partial \tau_x > 0$. (D.8) therefore implies $\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}} < 0$. The same exercise gives the required result that $\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}} = 0$ when $\nu = 1$ and $\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}} > 0$ when $\nu < 1$. By (D.7), $\partial \tau_x^*/\partial p_x$ shares the same signs.

■

D.4 Proof of Proposition 3

For simplicity, denote $\tilde{\phi}(s) = \phi s$ as the maximum cost level faced by a household with s savings in village j throughout this proof, and the unit cost

$$c_{Mj} = \left(\alpha_j p_x^{1-\sigma} + (1-\alpha_j) w_{aj}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

We start with an additional lemma.

Lemma 5. *For any level of savings s, a household in village j chooses the modern technology if*

$$z \geq A_T \left(\frac{c_{Mj}}{w_{aj}}\right)^{\gamma}$$

Proof. Traditional profit is

$$\pi_T = \begin{cases} (p_j A_T)^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}} (1-\gamma), & \text{if } 1 \leq \frac{\tilde{\phi}^{1-\gamma} w^{\gamma}}{\gamma p T} \\ (p_j A_T) \left(\frac{\phi}{w}\right)^{\gamma} - \phi, & \text{otherwise} \end{cases}$$

and modern profit is

$$\pi_M(z_M) = \begin{cases} (1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}} (p_j z)^{\frac{1}{1-\gamma}} c_M^{\frac{-\gamma}{1-\gamma}}, & \text{if } z_M \leq \overline{z}^M \\ p_j z \tilde{\phi}^{\gamma} c_M^{-\gamma} - \tilde{\phi}, & \text{otherwise} \end{cases}$$

where c_M is the unit cost function $c_M = (\alpha p_x^{1-\sigma} + (1-\alpha) w^{1-\sigma})^{\frac{1}{1-\sigma}}$ and the cut-off value for the financial friction is $\overline{z}^M = \tilde{\phi}^{1-\gamma} c_{Mj}^{\gamma} p_j^{-1} \gamma^{-1}$.

The main step here is to show that the cutoff value for using modern technology z^* does not depend on which constraints are binding. First, we look for a cutoff z^* if the financial friction does not bind for both traditional and modern at z^*.

$$
\begin{aligned}
(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}}(p_j z)^{\frac{1}{1-\gamma}} c_{Mj}^{\frac{-\gamma}{1-\gamma}} &\geq (p_j A_T)^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_{aj}}\right)^{\frac{\gamma}{1-\gamma}} (1-\gamma) \\
\implies \quad z &\geq A_T \left(\frac{c_{Mj}}{w_{aj}}\right)^{\gamma}
\end{aligned}
$$

Similarly, if financial frictions are binding at the cutoff z^* for both technologies,

$$
\begin{aligned}
pz\tilde{\phi}^{\gamma} c^{-\gamma} - \tilde{\phi} &\geq (pA_T)\left(\frac{\tilde{\phi}}{w}\right)^{\gamma} - \tilde{\phi} \\
\implies \quad z &\geq A_T \left(\frac{c_{Mj}}{w_{aj}}\right)^{\gamma}
\end{aligned}
$$

The last step is to show that this is the only possible cutoff value. If at the cutoff value z^* we have a household financially constrained for the traditional technology but not the modern technology, the profit indifference condition is

$$
p_j A_T \tilde{\phi}^{\epsilon} w_{aj}^{-\gamma} - \tilde{\phi} = (1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}}(p_j z^*)^{\frac{1}{1-\gamma}} c_{Mj}^{\frac{-\gamma}{1-\gamma}}
$$

That the modern sector is unconstrained implies

$$
\begin{aligned}
z^* &\leq \overline{z}^M = \tilde{\phi}^{1-\gamma} c_{Mj}^{\gamma} p_j^{-1} \gamma^{-1} \\
\implies (z^*)^{\frac{1}{1-\gamma}} &\leq \tilde{\phi} c_{Mj}^{\frac{\gamma}{1-\gamma}} p_j^{\frac{-1}{1-\gamma}} \gamma^{\frac{-1}{1-\gamma}}
\end{aligned}
$$

Combining these two conditions yields

$$
\gamma p_j A_T w_{aj}^{-\epsilon} \leq \tilde{\phi}^{1-\epsilon}
$$

which is a contradiction of the fact that the household is constrained in the traditional technology. A nearly identical procedure implies that there can be no cut-off value for any household in which the household is constrained in the modern technology but not the traditional one. Thus,

$$
z^* = A_T \left(\frac{c_{Mj}}{w_{aj}}\right)^{\gamma}
$$

is the only feasible cutoff for any household in village j. ■

After this, the proof of Proposition 3 follows almost immediately, because we can rely on the usual properities of the Pareto distribution in the unconstrained profit maximizing case.

Proof of Proposition 3

Proof. By the properties of the Pareto distribution

$$S_{jt} = Pr(z > z_{jt}^*) = \left(\frac{\underline{z}_M}{z_{jt}^*}\right)^{\theta_m} \implies \Delta \log(S_{jt}) = -\theta \Delta \log(z_{jt}^*)$$

and

$$\mathbb{E}[\log(z)|\text{use modern}] = \log(z_{jt}^*) + \frac{1}{\theta_M} \implies \Delta\mathbb{E}[\log(z)|\text{use modern}] = \Delta \log(z_{jt}^*)$$

Combining these two relationships and averaging across villages gives the required result. ■

Appendix References

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